Selfish Cops and Adversarial Robber: Multi-Player Pursuit Evasion on Graphs

Ath. Kehagias and G. Konstantinidis

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April 3, 2017
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The generalization to $N$-player game is immediate.
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Some Notation

- **Player set** \( I = \{1, 2, 3\} \).
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Conjectures etc.

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- Total payoff is sum of turn payoffs.

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1 & \text{if } s \text{ not a capture state} \\
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- \( B \) is the noncapture penalty.
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- Each cop has motive to capture the robber; capture by the other cop is a partial loss.
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Clearly it is a *non-zero-sum* game.

So the appropriate solution concept is *Nash Equilibrium* (NE).
The Main Theorem

In three-player SCAR, for any starting state $s$ we have

$$\forall i \in I, \forall \sigma^i : Q_s^i(\pi^1, \pi^2, \pi^3) \geq Q_s^i(\sigma^i, \pi^{-i}).$$

(1)

In other words, $\pi = (\pi^1, \pi^2, \pi^3)$ is a Nash equilibrium for three-player SCAR with any starting state $s$. 
Theorem

\[ \Gamma_s(i) \text{ is the two-player zero-sum game with initial state } s, \text{ played by player } i \text{ (with payoff } Q^i) \text{ against the coalition of players } I \setminus \{i\} \text{ (with payoff } -Q^i). \]
**An Auxiliary Lemma**

- $\Gamma_s(i)$ is the *two-player zero-sum* game with initial state $s$, played by player $i$ (with payoff $Q^i$) against the *coalition* of players $I \setminus \{i\}$ (with payoff $-Q^i$).
- $\Gamma_s(i)$ is a two-player, zero-sum *positive stochastic* game.
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- \( \Gamma_s(i) \) is a two-player, zero-sum *positive* stochastic game.

**Lemma**

*For each \( s \) and \( i \), the game \( \Gamma_s(i) \) has a value and the players have deterministic and stationary optimal strategies.*
The value and optimal strategies can be computed by a value-iteration algorithm which reduces to a Hahn-like algorithm.
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Denote by $\phi^i$ the optimal (maxmin) strategy of player $i$ in $\Gamma_s(i)$ and by $\phi^i_{\neg i}$ the joint (optimal) strategy of the coalition $I/\{i\}$ against $i$. 
The Main Theorem Proof: the Central Idea

The threat strategy of player $i$ is denoted by $\pi^i$ and defined as follows:
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Note that:
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The threat strategy of player $i$ is denoted by $\pi_i$ and defined as follows:

1. as long as every player $j \neq i$ follows $\phi_j^i$, player $i$ follows $\phi_i^i$;

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Note that:

- If player $j$ deviates then the players in $I \setminus \{j\}$ play the coalition strategy optimal against $j$ in $\Gamma_s(j)$. 
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Note that:

- If player $j$ deviates then the players in $I\setminus\{j\}$ play the *coalition strategy* optimal against $j$ in $\Gamma_s (j)$.
- The deviation will be detected immediately, since the game has perfect information.
Remarks

1. \( \pi = (\pi^1, \pi^2, \pi^3) \) is a NE iff (for every \( i \)) player \( i \) has no incentive to \textit{unilaterally} deviate from strategy \( \pi^i \).
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5. The strategies \( (\pi^1, \pi^2, \pi^3) \) are not stationary.
The auxiliary two-player zero-sum games are $\Gamma_s(1), \ldots, \Gamma_s(N)$. 
$N$-player SCAR

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In other words, $\pi = (\pi^1, \pi^2, ..., \pi^N)$ is a Nash equilibrium for $N$-player SCAR with any starting state $s$. 
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Remark

These results can be extended to $N$-player generalized CR games.
1. **Conjecture** The NE of our Theorem is not subgame-perfect.

2. If $c(G) = 1$ then every NE results in capture.

3. If either 2 or 3 is false, characterize the graphs for which it is true.

4. What changes in the case of the win/lose game?

5. What changes in the case of the concurrent game?
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- In a *concurrent CR game*, all players move simultaneously.
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  - Two or $N$ players.
  - Qualitative or quantitative game.
  - Generalized CR games (ala Bonato+MacGillivray).
  - Etc.
- For the two-player, zero-sum quantitative game of unselfish cops see Kehagias+Konstantinidis, TCS, vol.645, pp.48-59.
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We have results for a simplified case: selfish cops and passive robber.
The game is played between two cops; each tries to capture the robber first.
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Selfish Cops and Passive Robber

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- Payoff:
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Payoff:

- $C_1$ wants to maximize probability of capturing the robber.
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Payoff:

- \( C_1 \) wants to maximize probability of capturing the robber.
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Payoff:

- $C_1$ wants to maximize probability of capturing the robber.
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For the sequential case, and with deterministic robber, the payoff takes values in $\{0, 1\}$. 
Results for concurrent variant.
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- The game has value, optimal strategy for $C_1$, $\epsilon$-optimal strategy for $C_2$. 
Selfish Cops and Passive Robber

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These results can be extended to generalized CR games.
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