Selfish Cops and Adversarial Robber: Multi-Player Pursuit Evasion on Graphs

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April 3, 2017

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- The generalization to N-player game is immediate.

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- B is the noncapture penalty.

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- Each cop has motive to capture the robber; capture by the other cop is a partial loss.
- Clearly it is a *non-zero-sum* game.
- So the appropriate solution concept is *Nash Equilibrium* (NE).

The Main Theorem

Theorem

In three-player SCAR, for any starting state s we have

$$\forall i \in I, \forall \sigma^i : Q_s^i(\pi^1, \pi^2, \pi^3) \geq Q_s^i(\sigma^i, \pi^{-i}). \qquad (1)$$

In other words, $\pi = (\pi^1, \pi^2, \pi^3)$ *is a Nash equilibrium for three-player SCAR with any starting state s.*

An Auxiliary Lemma

- $\Gamma_s(i)$ is the *two-player zero-sum* game with initial state s, played by player i (with payoff Q^i) against the *coalition* of players $I \backslash \{i\}$ (with payoff $-Q^i$).

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Lemma

For each s and i, the game $\Gamma_s(i)$ has a value and the players have deterministic and stationary optimal strategies.

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- Denote by ϕ_i^i the optimal (maxmin) strategy of player i in $\Gamma_s(i)$ and by ϕ_i^{-i} the joint (optimal) strategy of the coalition I/i against i.

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- If player j deviates then the players in $I \backslash \{j\}$ play the *coalition strategy* optimal against j in $\Gamma_s(j)$.
- The deviation will be detected immediately, since the game has perfect information.

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Remark

These results can be extended to N-player generalized CR games.

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- For the two-player, zero-sum quantitative game of unselfish cops see Kehagias+Konstantinidis, TCS, vol.645, pp.48-59.

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- We have results for a simplified case: *selfish cops and passive robber*.

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  - For the sequential case, and with deterministic robber, the payoff takes values in $\{0, 1\}$.

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