How to Hunt an Invisible Rabbit on a Graph

Tatjana V. Abramovskaya[1] Fedor V. Fomin[2]
Petr A. Golovach[2] Michał Pilipczuk[3]

[1]St.-Petersburg State University

[2]University of Bergen

[3]University of Warsaw

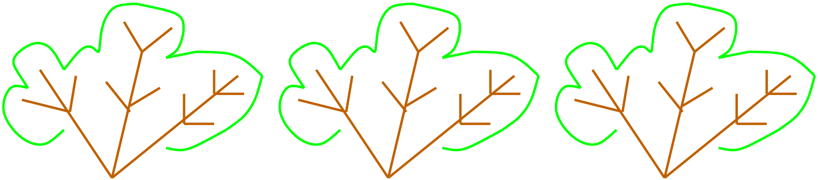
8th workshop on GRAph Searching, Theory & Applications,
Crete, Greece, April 13, 2017.

Outline

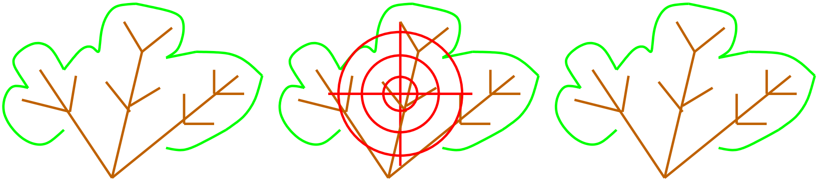
1. Introduction
2. Hunting rabbit on a grid
3. Conclusions

Hunters & Rabbit game

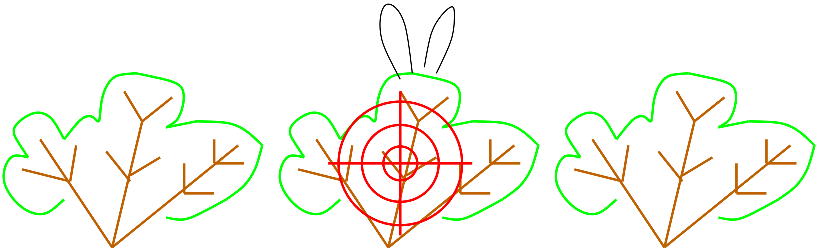
Hunters & Rabbit game



Hunters & Rabbit game

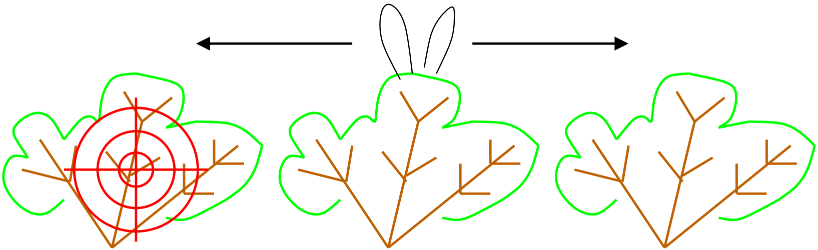


Hunters & Rabbit game

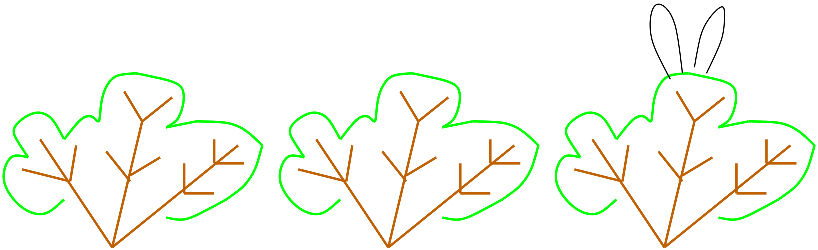


Hunters & Rabbit game

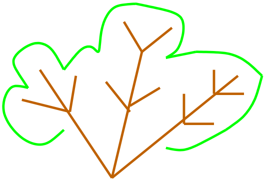
Hunters & Rabbit game



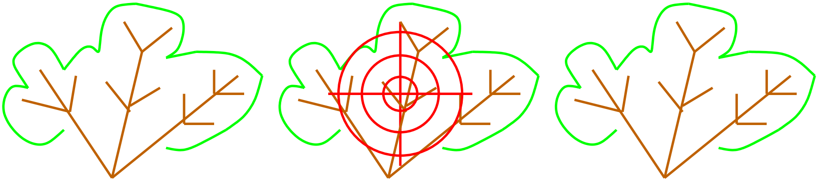
Hunters & Rabbit game



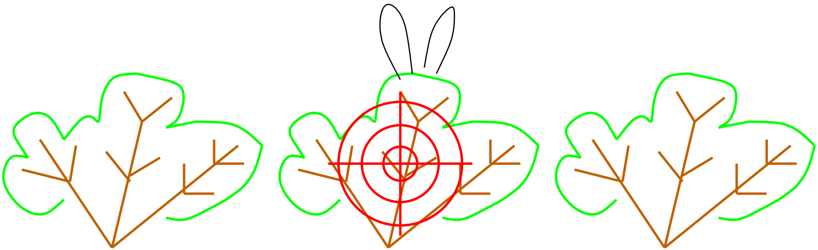
Hunters & Rabbit game



Hunters & Rabbit game

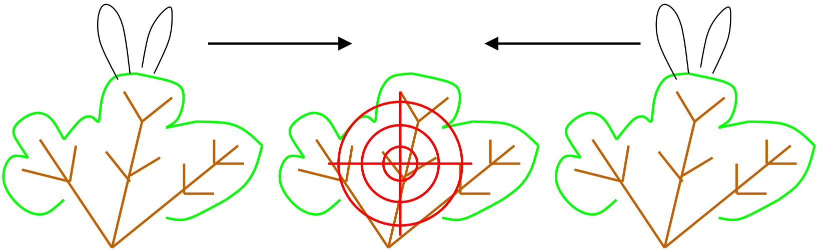


Hunters & Rabbit game

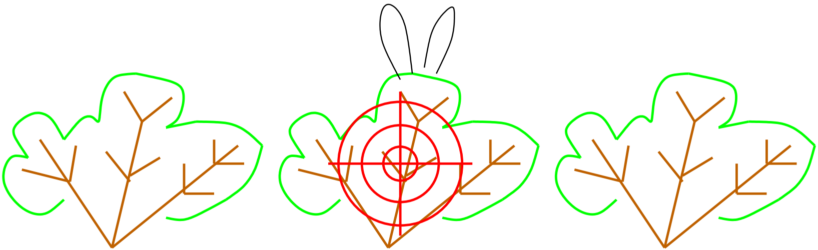


Hunters & Rabbit game

Hunters & Rabbit game

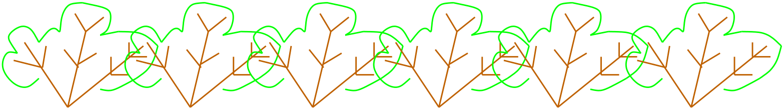


Hunters & Rabbit game

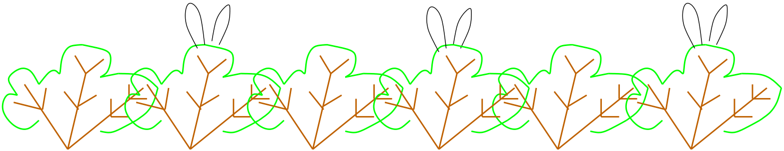


Hunters & Rabbit game

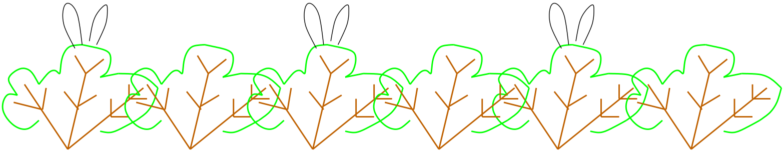
Hunters & Rabbit game



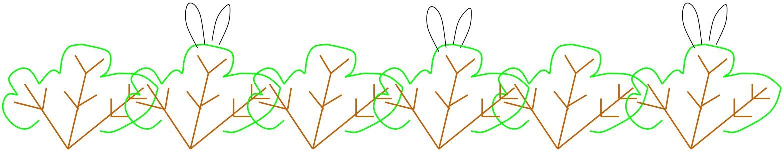
Hunters & Rabbit game



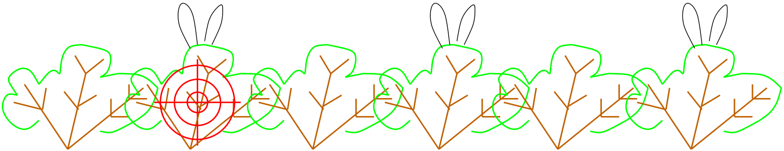
Hunters & Rabbit game



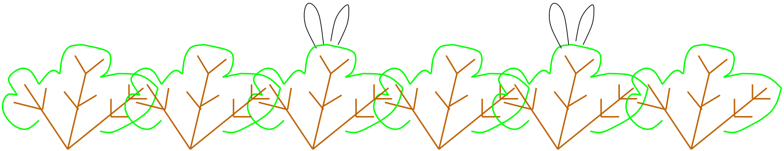
Hunters & Rabbit game



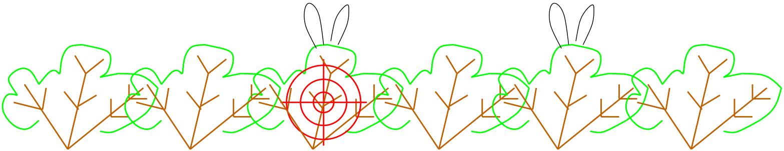
Hunters & Rabbit game



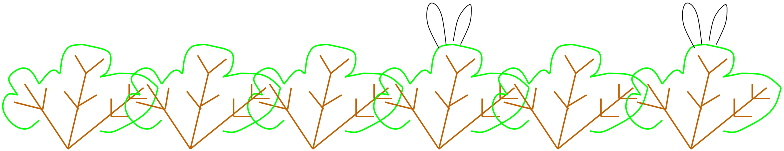
Hunters & Rabbit game



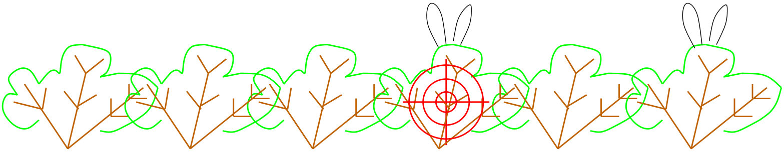
Hunters & Rabbit game



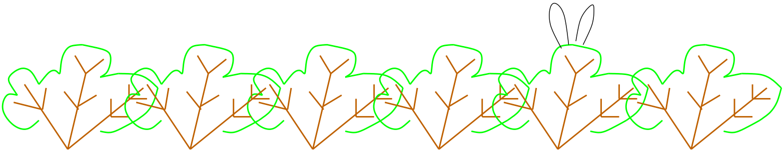
Hunters & Rabbit game



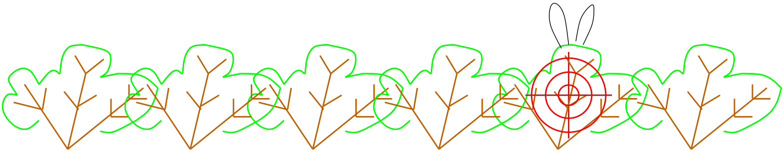
Hunters & Rabbit game



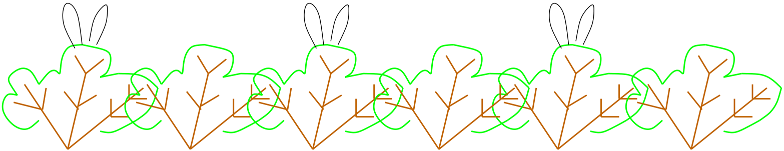
Hunters & Rabbit game



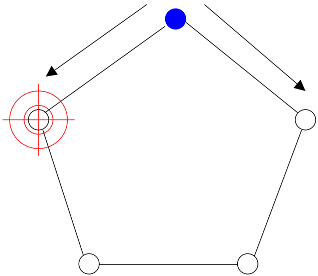
Hunters & Rabbit game



Hunters & Rabbit game



Hunters & Rabbit game



Hunters & Rabbit game

The Hunters & Rabbit game is played by two players, Hunter and Rabbit on an undirected graph.

Hunters & Rabbit game

The Hunters & Rabbit game is played by two players, Hunter and Rabbit on an undirected graph.

Hunter player has a team of hunters who attempt to shoot the rabbit.

Hunters & Rabbit game

The Hunters & Rabbit game is played by two players, Hunter and Rabbit on an undirected graph.

Hunter player has a team of hunters who attempt to shoot the rabbit.

At the beginning of the game, Rabbit player selects a vertex and occupies it, then the players take turns starting with the Hunter player.

Hunters & Rabbit game

The Hunters & Rabbit game is played by two players, Hunter and Rabbit on an undirected graph.

Hunter player has a team of hunters who attempt to shoot the rabbit.

At the beginning of the game, Rabbit player selects a vertex and occupies it, then the players take turns starting with the Hunter player.

At every round of the game each of the hunters selects some vertex of the graph and the hunters shoot simultaneously at their respective aims.

Hunters & Rabbit game

The Hunters & Rabbit game is played by two players, Hunter and Rabbit on an undirected graph.

Hunter player has a team of hunters who attempt to shoot the rabbit.

At the beginning of the game, Rabbit player selects a vertex and occupies it, then the players take turns starting with the Hunter player.

At every round of the game each of the hunters selects some vertex of the graph and the hunters shoot simultaneously at their respective aims.

If the rabbit is not in a vertex that is hit by a shot, it jumps to an adjacent vertex.

Hunters & Rabbit game

The Hunters & Rabbit game is played by two players, Hunter and Rabbit on an undirected graph.

Hunter player has a team of hunters who attempt to shoot the rabbit.

At the beginning of the game, Rabbit player selects a vertex and occupies it, then the players take turns starting with the Hunter player.

At every round of the game each of the hunters selects some vertex of the graph and the hunters shoot simultaneously at their respective aims.

If the rabbit is not in a vertex that is hit by a shot, it jumps to an adjacent vertex.

The hunter number of a graph G, denoted by $h(G)$, is the minimum number of hunters required to win in the Hunters & Rabbit game on G.

Hunters & Rabbit game

Sergiu Hart, Personal communication during a dinner in the Conference Centre “De Werelt” in Lunteren, the Netherlands.

Hunters & Rabbit game

SERGIU HART, Personal communication during a dinner in the Conference Centre "De Werelt" in Lunteren, the Netherlands.

J. R. BRITNELL AND M. WILDON, *Finding a princess in a palace: a pursuit-evasion problem*, Electron. J. Combin., 20 (2013), pp. Paper 25, 8.

Hunters & Rabbit game

Sergiu Hart, Personal communication during a dinner in the Conference Centre "De Werelt" in Lunteren, the Netherlands.

J. R. Britnell and M. Wildon, *Finding a princess in a palace: a pursuit-evasion problem*, Electron. J. Combin., 20 (2013), pp. Paper 25, 8.

R. Fedorov, A. Belov, A. Kovaldzhi, I. Yashchenko, A. Kovaldzh *Moscow Mathematical Olympiads, 2000-2005 (MSRI Mathematical Circles Library)*, AMS & MSRI, 2011.

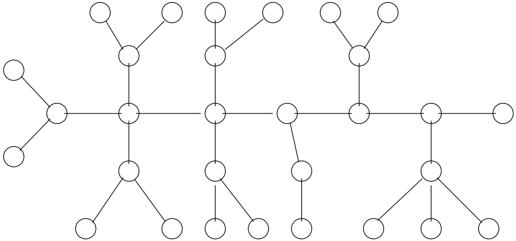
Hunters & Rabbit game

The stage of a video game is made up of round shelters connected by tunnels; a possible configuration is shown in the illustration. Your target is in one of the shelters, but you cannot see it. You can blast one shelter at a time, and if its the one where your target is, you win. In between shots the target must cross a single tunnel into a neighboring shelter; it doesnt matter if that shelter has been hit before. You have a winning strategy if you can plan a sequence of shots that will eventually hit the target no matter where it starts and what moves it makes.

(a) Prove that the configuration in the figure does not admit a winning strategy.

(b) Find all configurations that do not admit a winning strategy, yet acquire one as soon as any of their tunnels is blocked.

Hunters & Rabbit game



Our results

- $h(G) = \lfloor \frac{\min\{n,m\}}{2} \rfloor + 1$ if G is an $(n \times m)$-grid.

Our results

- $h(G) = \lfloor \frac{\min\{n,m\}}{2} \rfloor + 1$ if G is an $(n \times m)$-grid.
- $h(T) = O(\log n)$.

Our results

- $h(G) = \lfloor \frac{\min\{n,m\}}{2} \rfloor + 1$ if G is an $(n \times m)$-grid.
- $h(T) = O(\log n)$.
- For every positive integer k there exists a tree T_k such that $|V(T_k)| = 2^{O(k \log k)}$ and $h(T_k) \geq k$.

Hunting rabbit on a tree

V. Gruslys and A. Méroueh, *Catching a mouse on a tree* , CoRR abs/1502.06591 (2015).

Hunting rabbit on a tree

V. Gruslys and A. Méroueh, *Catching a mouse on a tree* , CoRR abs/1502.06591 (2015).

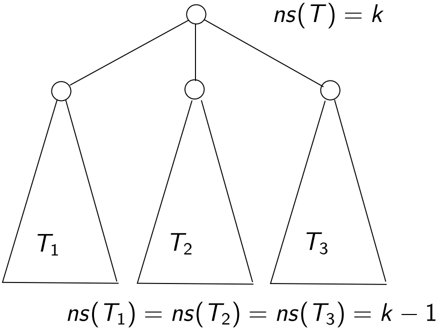
- $h(T) \leq \lceil (1/2) \log n \rceil$.

Hunting rabbit on a tree

V. Gruslys and A. Méroueh, *Catching a mouse on a tree* , CoRR abs/1502.06591 (2015).

- $h(T) \leq \lceil (1/2) \log n \rceil$.
- For any $\varepsilon > 0$ and any sufficiently large n, there is a tree T of order n such that $h(T) \geq (1/4 - \varepsilon) \log n$.

Hunting rabbit on a tree

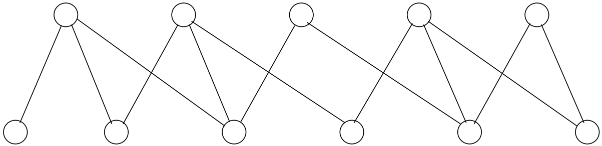


Hunting rabbit on a grid

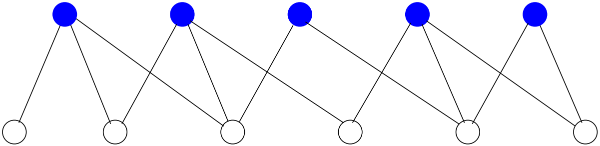
Theorem

Let G be an $(n \times m)$-grid. Then $h(G) = \lfloor \frac{\min\{n,m\}}{2} \rfloor + 1$.

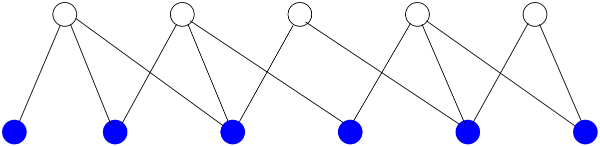
Hunting rabbit on a bipartite graph



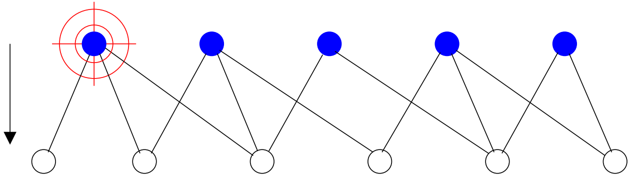
Hunting rabbit on a bipartite graph



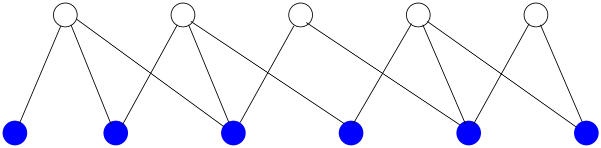
Hunting rabbit on a bipartite graph



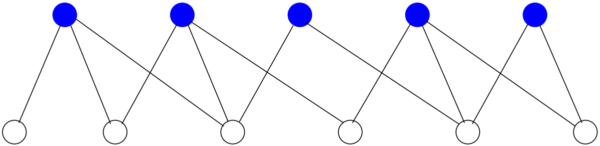
Hunting rabbit on a bipartite graph



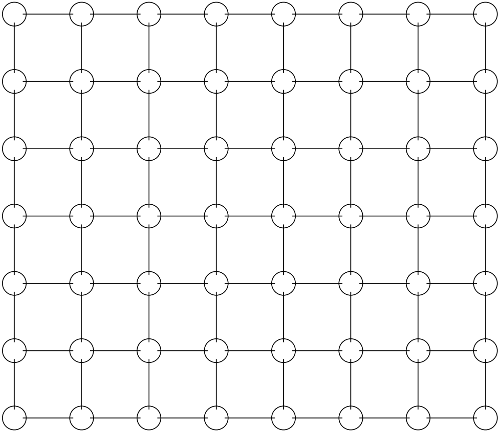
Hunting rabbit on a bipartite graph



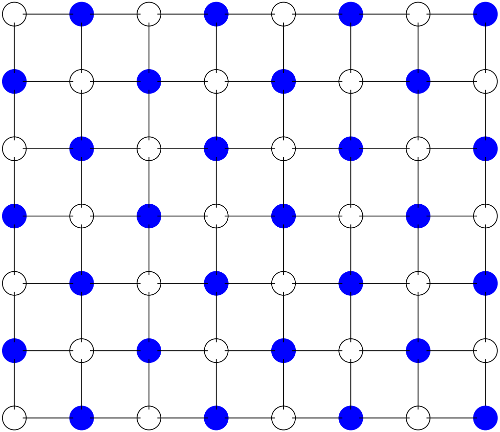
Hunting rabbit on a bipartite graph



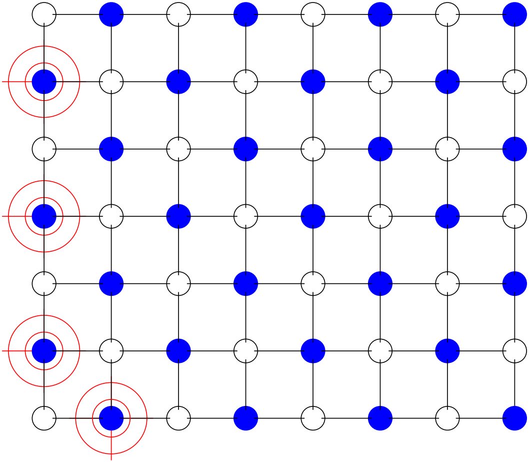
Upper bound



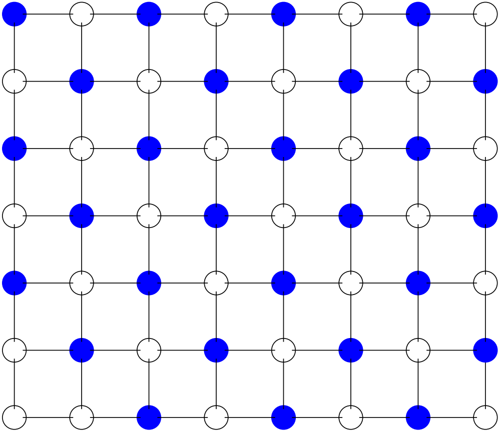
Upper bound



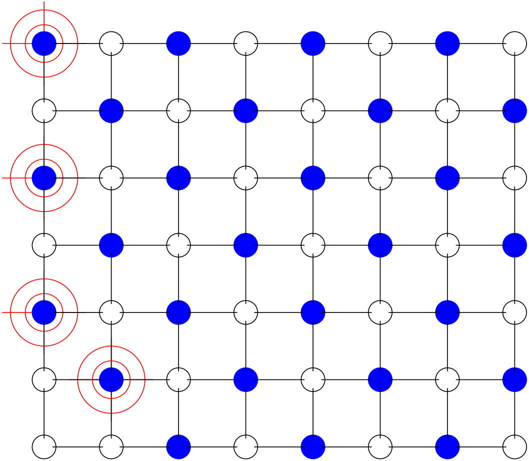
Upper bound



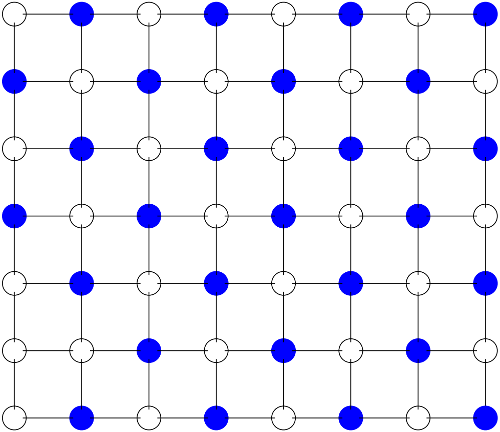
Upper bound



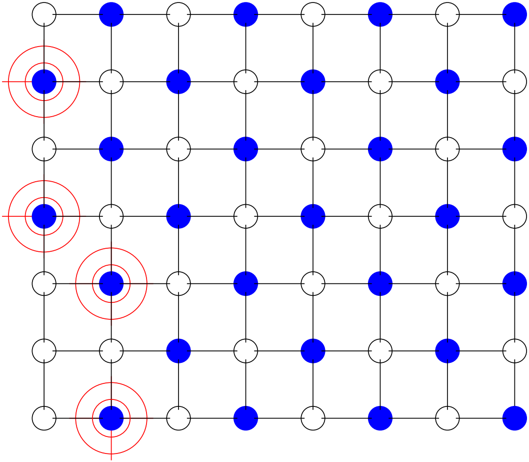
Upper bound



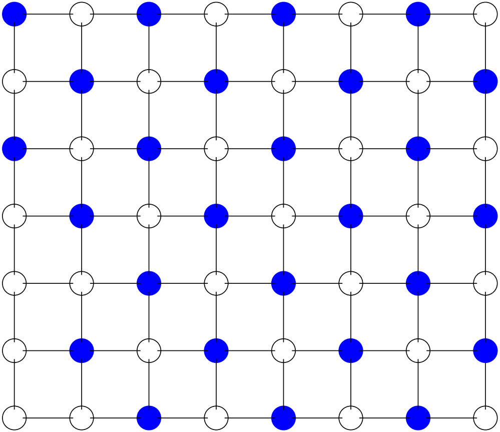
Upper bound



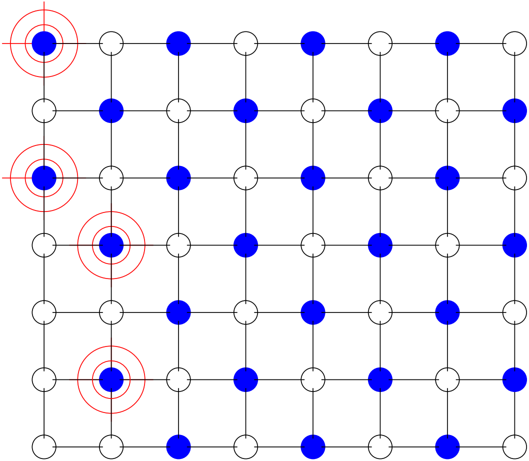
Upper bound



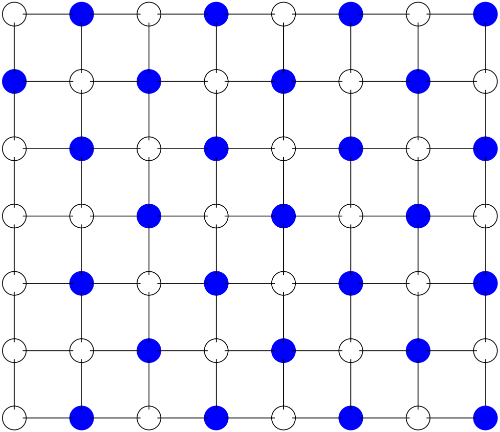
Upper bound



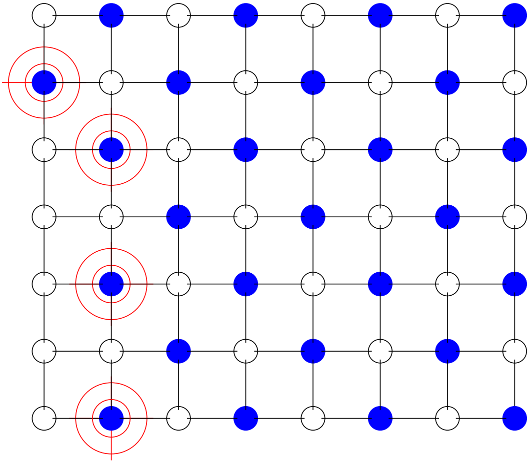
Upper bound



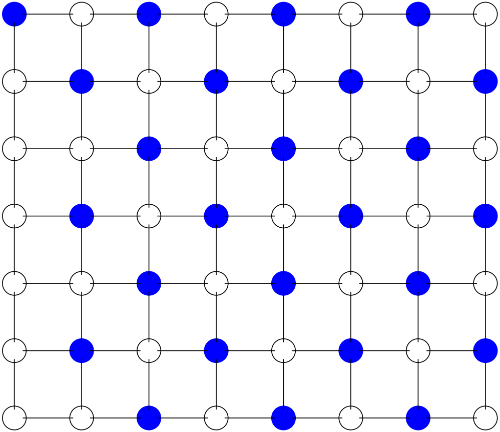
Upper bound



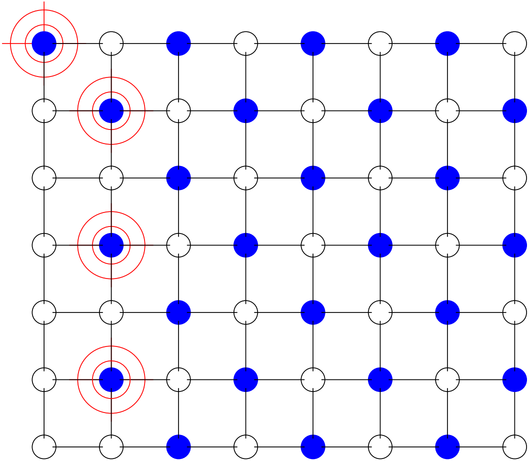
Upper bound



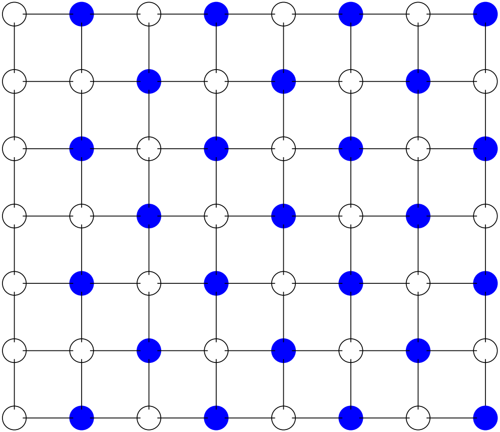
Upper bound



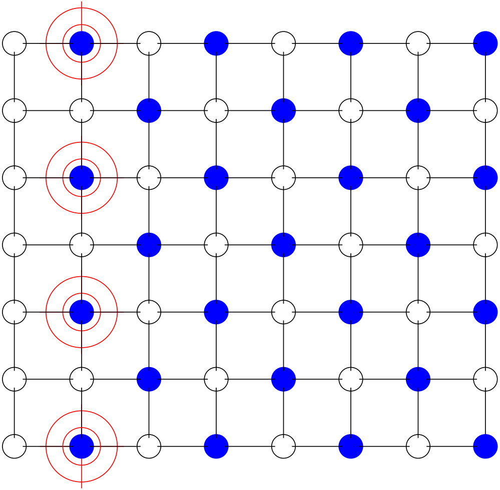
Upper bound



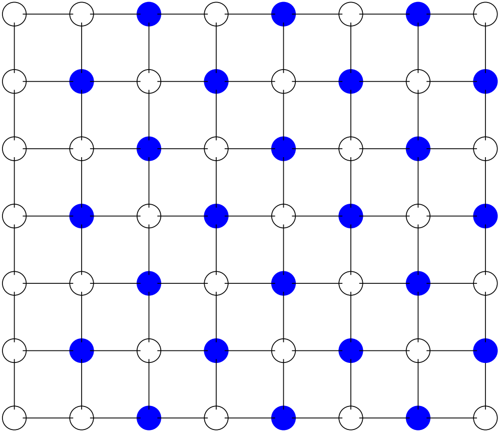
Upper bound



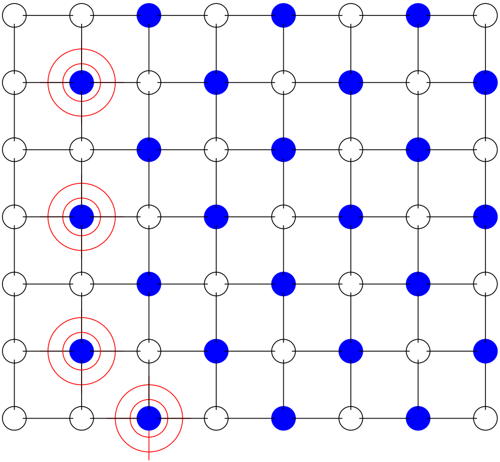
Upper bound



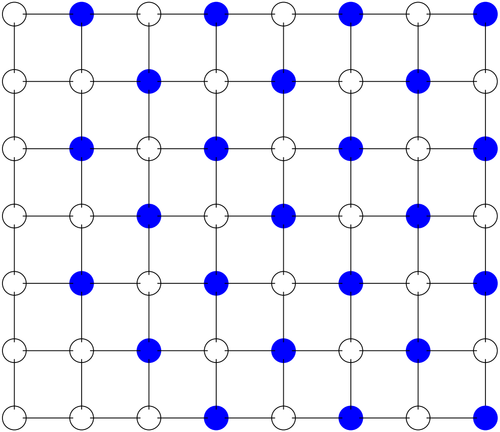
Upper bound



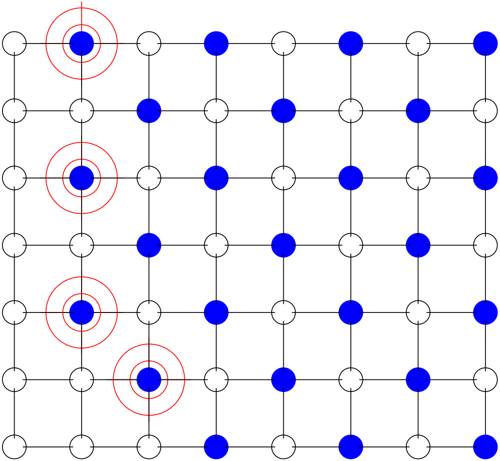
Upper bound



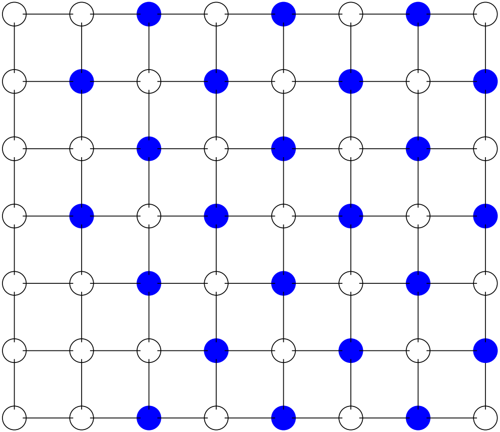
Upper bound



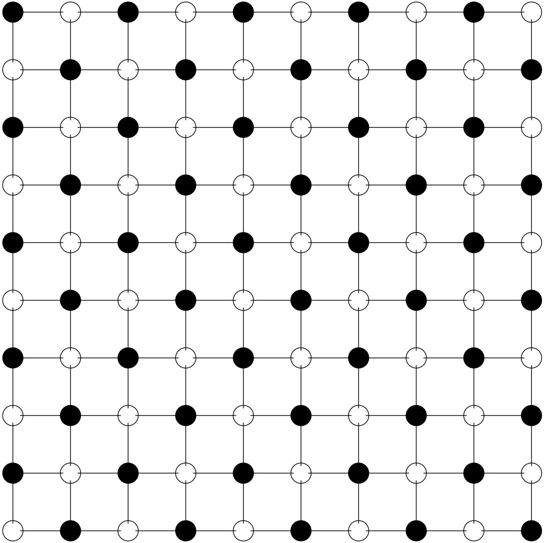
Upper bound



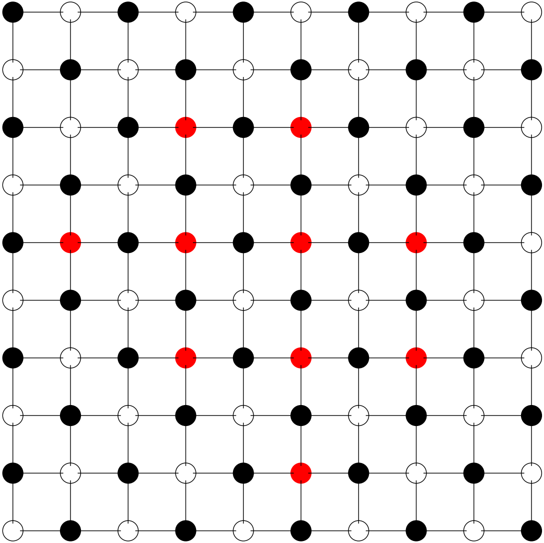
Upper bound



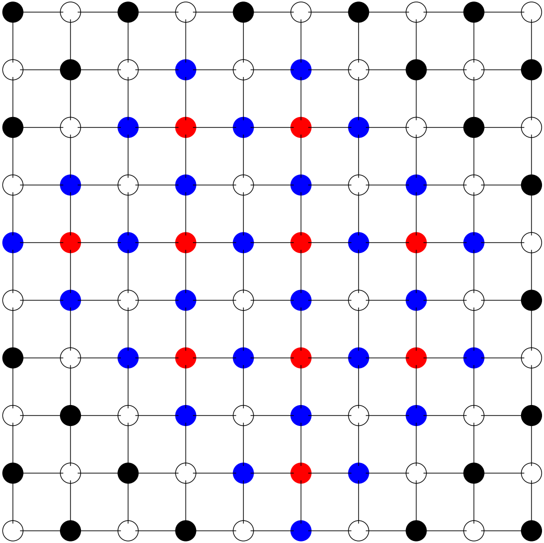
Isoperimetric theorem



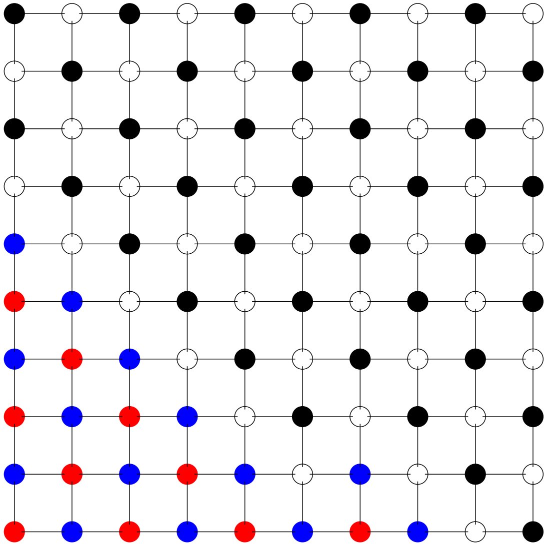
Isoperimetric theorem



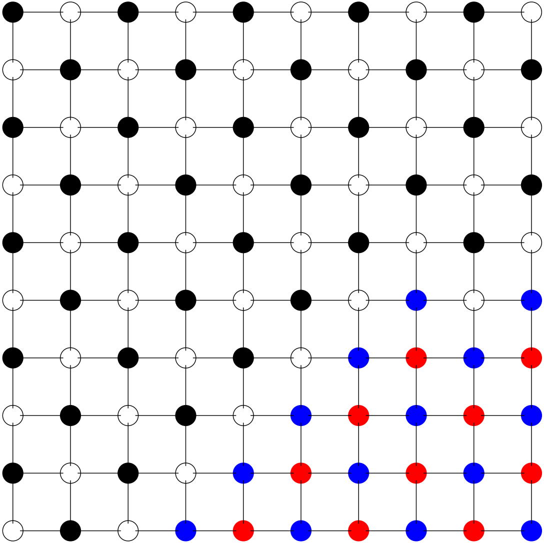
Isoperimetric theorem



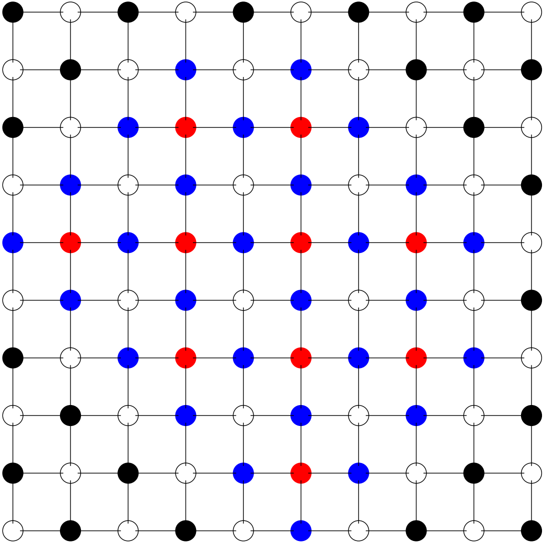
Isoperimetric theorem



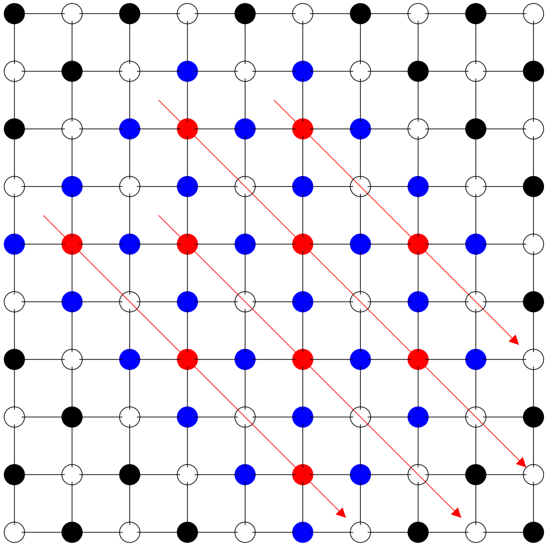
Isoperimetric theorem



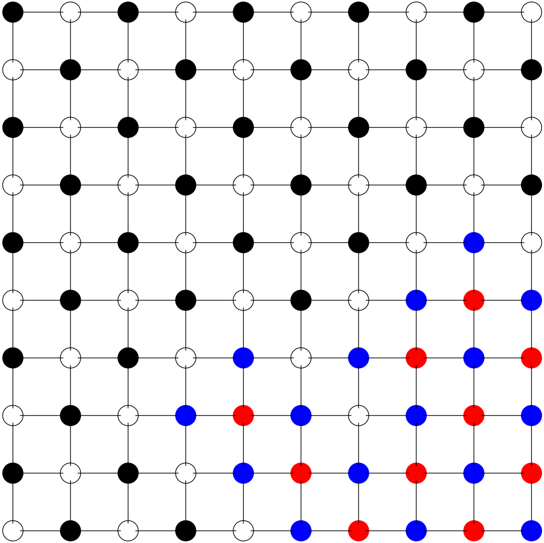
Isoperimetric theorem



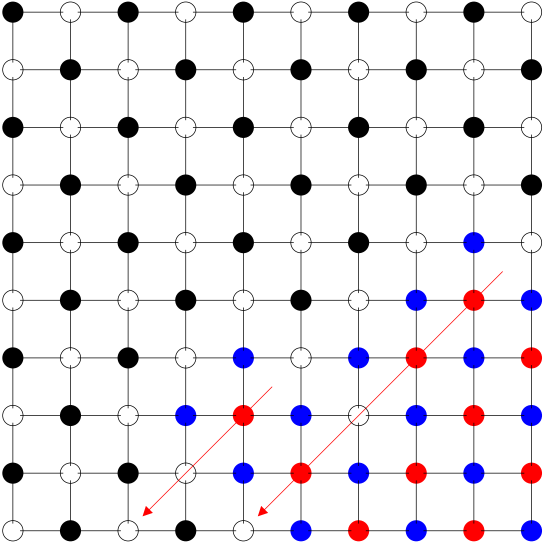
Isoperimetric theorem



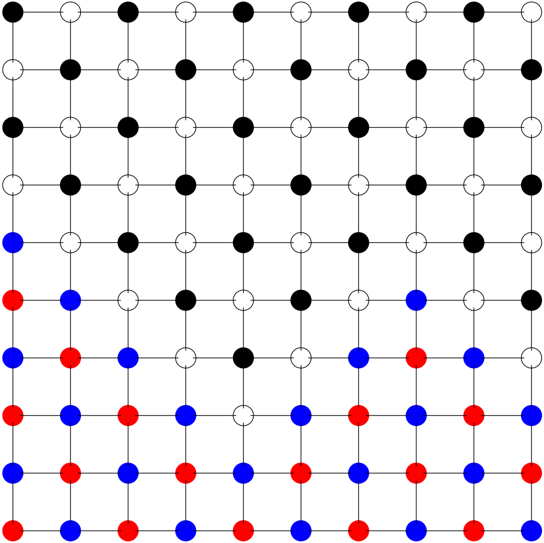
Isoperimetric theorem



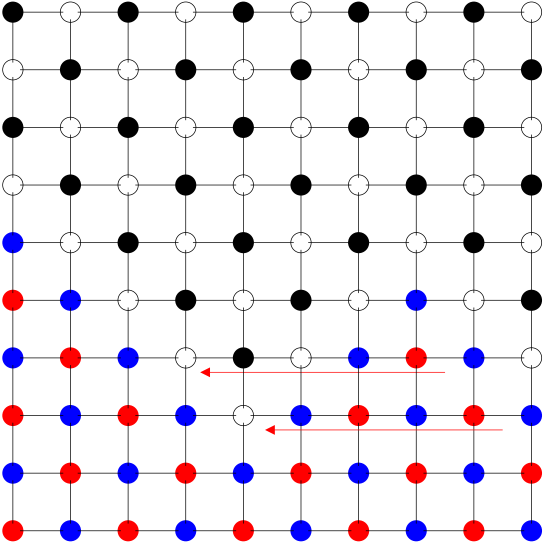
Isoperimetric theorem



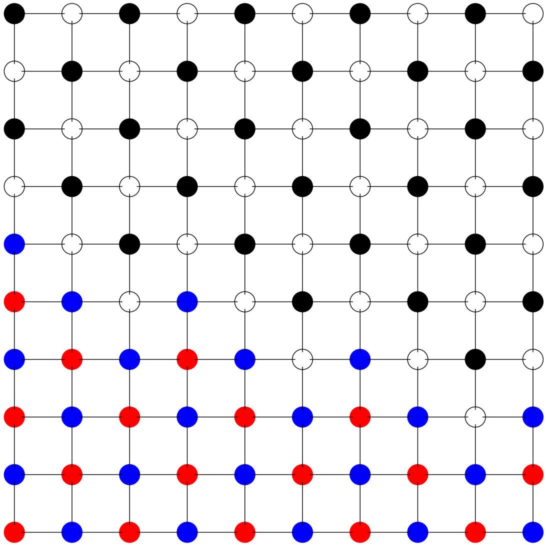
Isoperimetric theorem



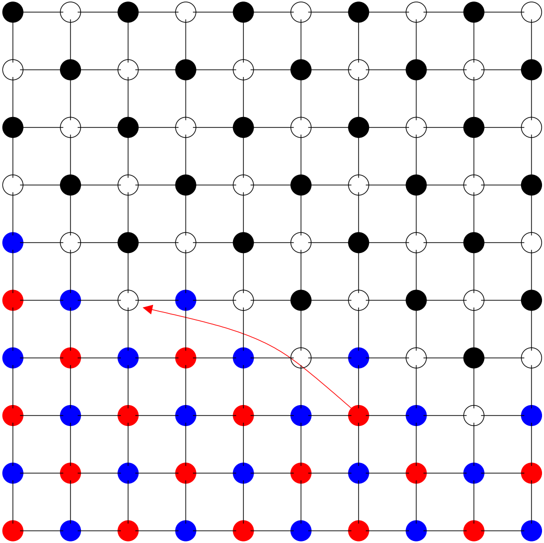
Isoperimetric theorem



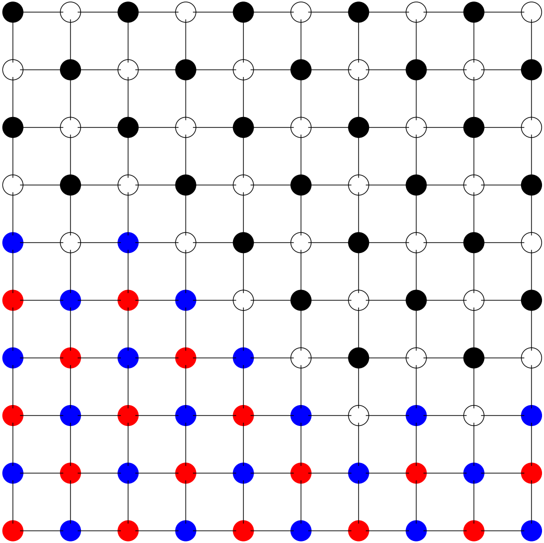
Isoperimetric theorem



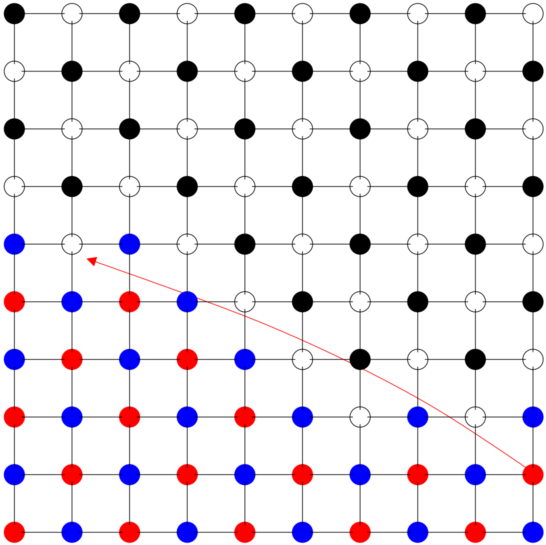
Isoperimetric theorem



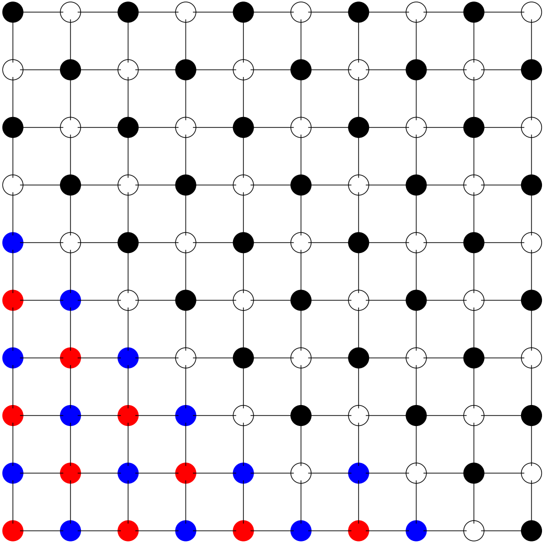
Isoperimetric theorem



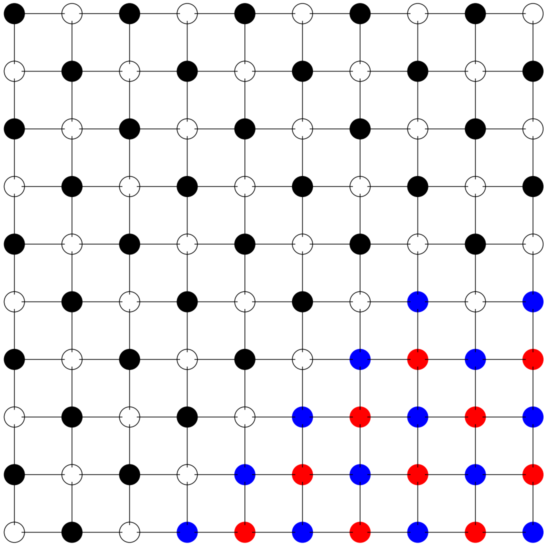
Isoperimetric theorem



Isoperimetric theorem



Isoperimetric theorem



Isoperimetric theorem

Corollary

Let G be an $(n \times n)$-grid, where $n \geq 4$ and n is even. Then for any $Q \subseteq V_1$ with $|Q| = \frac{n^2}{4} - \frac{n}{2}$, it holds that $\delta(Q) \geq \frac{n^2}{4}$.

Lower bound

- It is sufficient to show that $h(G) > n/2$ for $(n \times n)$-grid G, where $n \geq 4$ and n is even.

Lower bound

- It is sufficient to show that $h(G) > n/2$ for $(n \times n)$-grid G, where $n \geq 4$ and n is even.
- We show inductively that for every $i \geq 1$, each of the sets V_1 ("white set") and V_2 ("black set") has at least $\frac{n^2}{4}$ contaminated vertices after the i-th shot.

Lower bound

- It is sufficient to show that $h(G) > n/2$ for $(n \times n)$-grid G, where $n \geq 4$ and n is even.
- We show inductively that for every $i \geq 1$, each of the sets V_1 ("white set") and V_2 ("black set") has at least $\frac{n^2}{4}$ contaminated vertices after the i-th shot.
- Suppose that there is a set $X \subseteq V_1$ of contaminated vertices of size at least $\frac{n^2}{4}$ before the i-th shot. The hunters shoot to at most $n/2$ vertices of X. Hence, there is a set $Q \subseteq X$ of contaminated vertices of size $\frac{n^2}{4} - \frac{n}{2}$ such that the hunters do not shoot at them. Then at least $\frac{n^2}{4}$ vertices of V_2 are contaminated after the i-th shoot, because $\delta(Q) \geq \frac{n^2}{4}$.

Open problems

- Is it NP-hard or PSPACE-hard to compute the hunter number?

Open problems

- Is it NP-hard or PSPACE-hard to compute the hunter number?
- The graphs with $h(G) = 1$ can be recognized in polynomial time. Is deciding whether $h(G) \leq 2$ can be done in polynomial time?

Open problems

- Is it NP-hard or PSPACE-hard to compute the hunter number?
- The graphs with $h(G) = 1$ can be recognized in polynomial time. Is deciding whether $h(G) \leq 2$ can be done in polynomial time?
- Can the hunter number of a tree be computed in polynomial time?

HUCKLEBERRY FINN.

Thank You!