REFORM

The value of mathematics

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June 2008

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Reform

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Executive summary

The UK's maths economy which powers the financial services sector and wider industry is in danger of atrophy as fewer students study mathematics and attainment falls. At the core of this problem has been the diminution of the O-level/GCSE which has gone from being a key "staging post" to a "tick-box test". Scores of less than 20 per cent on the top paper regularly suffice to gain a grade C, despite a much reduced level of difficulty. Many students are turned off by the narrow teaching which results, and this has led to a generation of "lost mathematicians". Individuals lacking mathematical skills stand to lose £136,000 in income over a lifetime, and so have cost an estimated £9 billion to the UK economy since 1990.

Reform has conducted an analysis of O-level/GCSE examinations over time. From 1951 to 1970 these were a rigorous test of thought and initiative in algebra, arithmetic and geometry. Students were required to think for themselves. By 1980 questions were becoming simpler. Following the introduction of the GCSE there was a sharp drop in difficulty, with questions leading pupils step by step to a solution. Pass marks were lowered throughout the period.

The Gordian knot of political control has been tightened in an attempt to reverse the misguided trend towards "progressive" teaching. The unintended consequences of politicisation and centralisation of the subject are demotivation of teachers, a diminution of the enjoyment in mathematics by pupils and an exclusion of universities and employers from education policy. Steps to increase accountability taken by the Government and a focus on examination results have created unhelpful pressures on institutions and exam boards, which have in turn led to declining examination standards.

Relevance has replaced rigour in the belief that this would make mathematics more accessible. At the same time high stakes assessment has reduced what should be a coherent discipline to "pick 'n mix", with pupils being trained to answer specific shallow questions on a range of topics where marks can be most easily harvested. Instead a strong sequential approach ought to be taken building up a robust toolkit of cognitive and problem solving skills. In the modern global economy, it is the combination of core techniques, flexible thinking, logic and initiative that will be critical to future success.

The global maths economy is driven by high personal capability, initiative and logical thought. The top 5-10 per cent of mathematics graduates in the financial services sector practice "power maths", modelling derivatives and understanding financial risk. These skills are at the pinnacle of the City hierarchy making their practitioners the new "Masters of the Universe".

Yet the UK, home of Turing, father of modern information technology, and numerous recent prize winners such as Atiyah and Wiles, is failing to generate sufficient quality mathematicians. Financial services are being forced to recruit a high proportion of overseas graduates – as many as seven out of eight of all such posts. UK workplaces are finding themselves short of people with basic mathematics skills. Universities are being asked to select from a significantly reduced pool of applicants, a large number of whom are independently educated or from overseas.

Winning the battle of the maths economy will be critical to the UK's future success. Current Government policy is too small scale to deal with the pressing nature of the problem. Radical measures have to be taken to move mathematics from "geek to chic". Rigour must be central to this approach. The Government should step in and reverse the current inexorable drift towards modularising GCSE mathematics. A new Alexander is needed to cut the Gordian knot of state control and open up individuals' and institutions' ability to improve their own capability in the subject.

1
The battle for the maths economy

> "Mathematics, rightly viewed, possesses not only truth, but supreme beauty – a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show."
> **Bertrand Russell, The Study of Mathematics: Philosophical Essays**[1]

The 21st century economy is a maths economy. With increasing globalisation and a decline in manual work, almost all jobs rely on not only basic mathematics but also on logical thinking and cognitive ability. Mathematical skills are particularly important for the UK, where financial services make up 10 per cent of the economy, and where we are increasingly dependent on information technology.[2] The UK has historically had a strong record of producing first-class mathematicians. However, with key international competitors – from France to India – threatening to monopolise "the maths economy", the UK must ensure that it delivers at this crucial time.

The mathematics mind

Mathematics is a highly important discipline in its own right. It develops the fundamental skills of logical and critical reasoning, training the mind to be highly analytical and to deal with complex problems. It provides the basic language, structures and theories for understanding the world around us. In an increasingly technical world, the need to have these skills is ever increasing. Indeed, in the last 50 years, it is this mathematical language which has been increasingly used to control our environment and to organise commerce and society.[3]

A basic business requirement

Mathematics is one of the core requirements for almost all jobs.[4] Even relatively "unskilled" jobs would expect the candidate applying to have at least a grade C in mathematics. Since 1986 there has been a fall from 38 per cent of jobs requiring no qualification to only 28 per cent in 2006.[5] According to the CBI, more than 2.4 million employees with engineering and technical skills will be needed by 2014.[6] With an increasingly shifting focus on and towards climate change the power industry will require a flow of mathematicians as well as scientists and engineers to continue to deliver projects that are beginning to take shape now. The leading UK energy firm RWE announced a £1.7 billion investment programme in new cleaner gas and wind power, including offshore wind.[7] Further, the Government has committed itself to expanding nuclear power without addressing the long-term planning needed to generate the necessary home-grown technically competent manpower.

Further mathematics qualifications also benefit individuals in the workplace. Alison Wolf, Professor of Public Sector Management at King's College, London, has highlighted the economic returns on post-16 mathematics qualifications: "Even after allowing for every other factor imaginable, people who took A-level mathematics earn substantially more – around 10 per cent more – than those who did not."[8] This represents approximate additional earnings of £136,000 over a lifetime.[9]

Academic leadership

The UK has a strong record in producing first-class mathematicians, from Alan Turing, the father of modern computing and IT, to recent winners of international mathematics prizes including the prestigious Abel prize (Atiyah, 2004), two Fields' Medallists (Borcherds and Gowers, 1998), the Wolf Prize (Wiles, 1995) and the

1 Russell, B. (1910), *Philosophical essays*.
2 International Financial Services London (2008), *International financial markets in the UK.*
3 Adrian Smith in his official study *Making mathematics count* highlights the intrinsic value of mathematics: "Mathematics provides a powerful universal language and intellectual toolkit for abstraction, generalization and synthesis. It is the language of science and technology. It enables us to probe the natural universe and to develop new technologies that have helped us control and master our environment, and change societal expectations and standards of living. Mathematical skills are highly valued and sought after. Mathematical training disciplines the mind, develops logical and critical reasoning, and develops analytical and problem solving skills to a high degree." Smith, A. (2004), *Making mathematics count.*
4 Confederation of British Industry (2006), *Working on the three Rs: Employers' priorities for functional skills in mathematics and English*. The report emphasises the importance of functional numeracy in the workforce. Skills such as basic mental arithmetic, the ability to interpret and respond to quantitative data, calculating and understanding percentages and basic understanding of probability are considered to be part of functional mathematical skills required in day-to-day business operations.
5 The Economic and Social Research Council (ESRC) Society Today (2007), *Skills used in British workplaces still rising*.
6 CBI (2007), *1k annual bursary for every science student would help reverse skills decline*.
7 Ibid.
8 Wolf, A. (2002), *Does education matter? Myths about education and economic growth*, pp.35-36. Evidence from the USA has also shown that the traditional core parts of the school curriculum are increasingly the most important focus. "Moreover, it again seems to be mathematics skills which matter most [in relation to the future earning power of the country]."
9 This is a *Reform* calculation based on average salary for higher earners and the 10 per cent premium for maths A-level approximated by Alison Wolf. See Bosanquet, N et al. (2008), *Shifting the unequal state: From public apathy to personal capability, Reform;* Wolf, A. (2002), *Does education matter? Myths about education and economic growth*, pp.35-36.

Shaw prize (Wiles, 2005). Such high level achievements are important insofar as they reflect a healthy national pyramid, which routinely generates a mass of competent citizens at its base, and large numbers of highly competent, technically qualified graduates. This capability helps develop the next generation of "home-grown" mathematics talent and forms the growth bed for much of the commercial "mathematics talent", facilitating the development of computing experts and practitioners of "power mathematics".

The new Masters of the Universe

The UK's strong record in producing world-class mathematicians has aided its ascension as one of the financial capitals of the world, where financial services make up 10.1 per cent of GDP.[10]

About 40 per cent of mathematics graduates enter financial services.[11] Another 25 per cent or so may be classified as working in areas related to computing. A further 25 per cent work in the area of management services and statistics – providing algorithms to manage complex systems, or to extract and analyse crucial data needed for management decisions. The vast majority of mathematics graduates entering the financial sector go into mid-ranking roles which require competence and strong numeracy skills.[12] While sophisticated mathematical analysis is not required for these roles, individuals must have strong capability in mathematics, namely cognitive ability and logic.

The top 5-10 per cent go into sophisticated mathematics modelling roles in the City (generally as traders). This "power maths" has been developed over the last 20 years, marking a shift from the trading of physical products to derivatives, and increasing the importance of mathematical skill to the City. Where in the 1980s, the deal makers were characterised in Tom Wolfe's Bonfire of the Vanities as "Masters of the Universe", mathematical modellers are the new "Masters of the Universe", driving value in the City.[13] The importance of getting this right is highlighted by the current credit crunch.

The global maths economy

The shift in the City of London reflects growing demand for high-level mathematical modelling skills across the globe. This is in a market where capability is king – logical thinking, problem solving and cognitive ability. Many countries have entered this market, of which the City has reached a position of global prominence. *Reform* authors argue in *Shifting the unequal state* that this capability will be critical to the UK's future economic success.[14]

Many countries are seeking pre-eminence in the global maths economy, in addition to traditional competitors such as France (where there are 160 mathematics graduates per million people, compared to 119 mathematics graduates per million people in the UK), the US and Singapore.[15] Further afield, China and India are producing higher and higher numbers of mathematics graduates. A third of larger firms are now recruiting from India, particularly from the Indian Institutes of Technology, and 25 per cent recruit from China.[16] A recent US paper by William Kennedy at Michigan Tech on the condition of mathematics in US high schools reports that, while the US and the UK are producing fewer mathematics and science graduates, foreign universities are producing record numbers.[17] According to the CBI, China is producing 300,000 graduates every year in science, technology, engineering and mathematics – three times the number coming through UK universities. India has 450,000 engineering undergraduates in the current academic year alone.[18]

The battle for mathematical ascendency

In May last year Jan Figel, European Commissioner for Education, told *The Times* that if Europe's top universities don't act quickly and modernise "we will see an uptake or overtake by Chinese or Indian universities" as they have plans to put a lot more funding into their higher education.[19] Martin Read, LogicaCMG's Chief Executive, has also said that Britain cannot compete on numbers with China and India. In regards to graduates he said "businesses will start to relocate if they can't find them in their own country".[20]

As the battle for mathematical ascendency continues, those with the strongest mathematical capability will succeed. Pre-eminence in mathematics will be a defining part of the UK's future economic success.

10 IFSL (2008), *International financial markets in the UK*.
11 Prospects.ac.uk
12 Mathematics graduates are highly sought after – and increasingly so. "Destination statistics for mathematics graduates reveal a fall in unemployment, supporting the idea that employers value the high level of numeracy these graduates can offer." Prospects (2008), *What do graduates do 2008?*, p.20.
13 Wolfe, T. (1987), *The Bonfire of the Vanities*.
14 Bosanquet, N. et al. (2008), *Shifting the unequal state: From public apathy to personal capability, Reform*.
15 OECD StatsExtract, *Dataset: Graduates by field of education*.
16 CBI (2008), *Learning and skills news*.
17 Kennedy, W. (2007), *Teaching at tech: Bill Gates on re-engineering high schools*.
18 *The Guardian* (2006), "UK looking overseas for science graduates", 15 March.
19 *The Times* (2007), "Asia threatens to knock British universities off the top table", 21 May.
20 Ibid.

2
The falling standard of mathematics: public examinations 1951-2006

The debate about standards in mathematics has been alive for some time. This can be seen by a quotation in the very first issue of the Journal of the Glasgow Mathematical Association, dated December 1936, about university teaching of mathematics and the influence on it by the preparation provided by school:

> "The weakness in the technique of algebraical manipulation is marked; and it is constantly necessary to digress from the theme at hand, to go back and explain some elementary algebraical principle involved, a digression which even ten or fifteen years ago would have been thought superfluous."[21]

Modern complaints often sound superficially similar, but are in fact of a different order. A team of mathematicians led by John Marks has conducted a thorough review of past examination papers, including archival material.[22] An assessment has been made of the standard of mathematics exam papers sat at age 16 in 1951, 1960, 1970, 1980, 1990, 2000 and 2006; these show that standards in mathematics have declined markedly over the last 28 years.[23] Age 16 has been chosen since GCSEs are nationally set exams taken by all pupils (in contrast, say, to SATs, which do not have to be taken by pupils in the independent sector). Further, GCSEs are often the minimum requirement for employment. O-level and GCSE cannot be strictly compared *in toto*. But such a comparison of O-level (1951-1987) and higher level GCSE (1987-2006) is instructive if restricted to the top 20-25 per cent, since it then focuses on what is expected of this key group, which contains all those now likely to study a numerate subject at university.

This report will examine four criteria in order to assess how standards have changed over time:

> Content
> Difficulty
> Style
> Pass standard

In assessing standards we have identified three phases:

> 1951, 1960 and 1970: standards remained broadly constant with a rigorous focus on algebra, arithmetic and geometry.
> 1980: the beginnings of a simplifying trend with greater emphasis on "mathematics in context", while the curriculum content remained formally the same as before.
> 1990, 2000 and 2006: steep decline in standards compared to previous years, with a curriculum that is both broader and much shallower.[24]

This chapter will analyse each phase in turn, showing how standards have changed.

21 Journal of Glasgow Mathematical Association (1936).
22 With thanks to the Cambridge Assessment Archives Service.
23 Professor Peter Tymms and Dr Robert Coe from the CEM Centre at the University of Durham have been at the forefront of this research. See Tymms, P., Coe, R. and Merrell, C. (2005), *Standards in English schools: changes since 1997 and the impact of government policies and initiatives*, CEM Centre, University of Durham. "From 1988 until 2004 the achievement levels have risen by about 1½ grades across all subjects on average. Exceptionally, from 1988 the rise appears to be about three grades for Mathematics." Their research is widely respected by schools, with around 200,000 students in over 1,200 secondary schools taking the YELLIS tests to predict GCSE scores. Yellis, the **YE**ar **11** **I**nformation **S**ystem, provides innovative tests widely used in the UK and elsewhere, forming a baseline for value added measures in secondary schools.
24 Mathematics exam papers from the following years were analysed: 1951 (the first year of O-level), 1960, 1970, 1980, 1990 (the fourth year of GCSE), 2000 and 2006 (the most recently available exam papers at the time of the study).

Content: broader and shallower

The evolution of the mathematics curriculum from a rigorous, focused discipline to a broader and shallower subject can be demonstrated by analysing exam papers since 1951, the year of the inception of the GCE O-level. Since then, and particularly since the inception of GCSEs in 1987, the content of mathematics exams has changed considerably. From being required to know arithmetic, algebra and geometry in depth during the years of O-level (1951-1987), candidates now have to exhibit a degree of familiarity with a much wider but shallower curriculum including handling data (statistics), vectors, transformations and using and applying mathematics.

1951, 1960, 1970, 1980: focus on arithmetic, algebra and geometry

O-levels represented a rigorous test of pupils' ability to understand and use mathematical concepts. Between 1951 and the inception of the GCSE in 1987, O-level candidates would be expected to sit papers which had questions involving arithmetic, algebra and geometry. This meant that candidates had to have a working knowledge of, and competence in, each of these fundamental areas of mathematics.[25] Formulae had to be memorised applicable to the area of the subject.

In 1951, 1960 and 1970 the most common form of examination was to set three separate papers entitled arithmetic, algebra and geometry.

Alternative papers of comparable difficulty were also available in which questions involving arithmetic, algebra and geometry were combined in an integrated way in the same paper; candidates usually sat two such papers.

By 1980 the former alternative paper became the most common form of examination once again retaining comparable syllabus content with previous years.

1990, 2000, 2006: a broader and shallower curriculum

GCSEs, in comparison, have a much broader and shallower curriculum.[26] In order to achieve a grade C, which was designed to be equivalent to a pass at O-level, candidates could get away with having a much weaker grasp on the fundamentals of mathematics.[27] Following the main classifications of the statutory National Curriculum, the questions covered the following topics: Number and Algebra; Shape, Space and Measures; Handling Data; and Using and Applying Mathematics.[28] The addition of new topics meant less of the core elements being included – algebra, arithmetic and geometry. New topics such as graphical representations, statistics and transformations were all covered, although not in a great degree of depth.

Difficulty and demand of questions has weakened

The difficulty and demand of questions has seen a gradual decline since the inception of O-levels in 1951. An analysis of mathematics O-level and GCSE papers between 1951 and 2006 shows this. There are three phases that can be identified: in 1951 to 1970 questions were rigorous and demanding; in 1980 to 1990, despite the content of the curriculum being identical to previous years, the simplifying trend was already taking place; and 1990 to present, when questions had became significantly shorter and simpler.

1951, 1960, 1970: rigorous and demanding questions

In 1951, 1960 and 1970 the questions in all three papers were all demanding; the compulsory questions were marginally more straightforward while the candidates were given a choice with the more difficult questions. Papers very similar to the 1951 papers were set in the subsequent 20-25 years; the 1960 and 1970 papers have been studied for this report.

- In arithmetic, the questions included serious multi-step problems which required appreciable conceptual depth of understanding together with accuracy and facility with numerical calculations.
- In algebra, the questions were nearly all multi-step problems requiring the ability to perceive the steps required in the solution together with accurate algebraic manipulation.

25 This provided them with a comprehensive grounding for any subsequent studies in these subjects and also in calculus, mechanics and other forms of applied mathematics, and coordinate geometry in two and three dimensions.
26 Gardiner, T. (2006), *Beyond the soup kitchen: Thoughts on revising the Mathematics "Strategies/Frameworks" for England*.
27 Grade C at GCSE was available by two routes – via the Higher Tier (Grades A-D) and the Intermediate Tier (Grades C-E). In each tier, candidates sat two papers. An alternative route involving 20 per cent coursework was also available.
28 These subdivisions follow the main classifications which ultimately defined the statutory National Curriculum for mathematics which came into force in the early 1990s.

- In geometry, the problems to be solved were multi-step and required a considerable knowledge of basic geometrical theorems and their use in constructing geometrical proofs. For example, in the 1951 geometry paper there are three compulsory questions requiring such proofs together with two more such questions in the optional part of the paper.

Pupils were required to master these three subjects in considerable depth and be able to answer complex multi-stage problems.

1980: beginnings of a simplifying trend

In 1980 the papers and questions had similar content to those for earlier years; however there is the clear beginning of a decline in difficulty. Fewer questions asked candidates to provide proof of concepts. Pupils were more led through the questions than in previous years, with signposting demonstrating the next phase, rather than being asked to generate the stages themselves.

1990, 2000, 2006: sharp decline in difficulty of questions

The GCSE papers, in comparison, contained shorter, more straightforward questions. The papers analysed contained less algebra and geometry. The focus moved from proving propositions to demonstrating results using graphs and other presentational devices.

The ability to solve complex, multi-step arithmetical or algebraic problems was virtually absent from the GCSE papers. The GCSE mathematics papers contained far fewer long, unstructured questions. Instead, questions were relatively straightforward and virtually all were broken down into separate simple stages so that the candidate was guided through the solution. This differed from the O-level papers, where candidates were asked to find his or her solution to what could have been set as a multi-step problem.

The problems were also straightforward with the solutions being well signposted. It would clearly be possible to have achieved a grade C pass without doing any significant amount of either algebra or classical Euclidean geometry or its equivalent.

Style of questions

Over the past 30 years, mathematics education has gone from expecting candidates to show initiative and understanding of mathematical concepts to being sign-posted through a problem with all necessary formulae given. In this sense, mathematics in exams has gone from a "conceptual" understanding of mathematics to the application of "rules". The style of questions has also changed from being abstract and conceptual to being embedded in artificial contexts, in a failed attempt to make everyday experiences the basis of the questions. Producing the answer became more a matter of following steps and presenting information, rather than showing initiative and thinking conceptually.

1951, 1960, 1970: focus on conceptual understanding

In the papers examined from 1951 to 1980, candidates were expected to prove propositions and show their conceptual understanding of mathematics. These questions related to mathematical concepts and rarely cited any practical application of the subject. The style of question was testing the ability to think logically and conceptually and to manipulate problems to reach the solution.

1980: move towards practical application

The papers examined in 1980 show that while the questions were of similar content, the practical application of the concepts had crept in at the expense of further manipulation of the concept. For example, the questions refer to a boat travelling across the water and ask pupils to draw a scale diagram to discover various distances and speeds.

1990, 2000, 2006: practical application is the focus

In the papers examined between 1990 and the present, the questions relied far more on "rules" than "proofs". Candidates were no longer expected to use their initiative and show their conceptual understanding of mathematics, but rather were signposted through questions to get to the answer.[29] The concept of proof was virtually absent in the GCSE papers, unlike O-level papers in which it was a central feature. GCSE candidates were asked to present answers graphically and in tables and there was a greater focus on presentation.

Pass standards

Pass standards have become much lower since the inception of GCSEs, in particular between 1990 and 2000. It is now possible to achieve a grade C in GCSE mathematics having almost no conceptual knowledge of mathematics. This is due in part to the simplicity of the questions and the decline of algebra, geometry and proof within the papers. It is also due to the decision following the Cockcroft report in 1982 to reward candidates for "what they can do" which has led to a system in which high grades are regularly obtained without getting questions completely correct.[30] Thus the undertaking given by Sir Keith Joseph when the GCSE was introduced has not been fulfilled – the standard of a GCSE grade C in mathematics is well below that of GCE O-level pass.

Indeed, it has become substantially easier to achieve a grade C since the inception of GCSEs in 1987.[31] It should be remembered that a grade C in GCSE was meant to be equivalent to a pass at O-level and is often the general minimum requirement for many employers.

> In 1990 the percentage mark on the Higher Tier for a grade C was just over 50 per cent. However, in 2000 and 2006 the required percentage mark for a grade C had fallen to about 20 per cent; this mark could be attained by answering correctly the first four questions on Paper 5 and Paper 6.

> It has also become easier to get a grade C in the Intermediate Tier. In 1990 the percentage mark on the Intermediate Tier for a grade C was 70 per cent. However, in 2000 and 2006 the required percentage mark for a grade C had fallen to just over 40 per cent.[32]

Standards for a GCSE grade C in mathematics are now lower than they were in 1990 despite some tightening of the regulations for formulae sheets and calculator use since then.

This is for a number of reasons which all have the effect of lowering the requirements for a grade C – or indeed other GCSE grades.

These are:

> considerably more restricted and less demanding syllabuses;

> less demanding style of questions in which candidates are no longer required to find their own way through two- three- or multi-stage questions;

> the availability of calculators in some papers;

> the availability of formulae sheets in all papers;

> the very low marks required in 2000 and 2006 for grade C pass.

29 London Mathematical Society (1995), *Tackling the mathematics problem*. "Compared with students in the early 1980s, there is a marked decline in students'analytical powers when faced with simple two-step or multi-step problems."
30 The Cockcroft Report (1982), *Mathematics counts*.
31 Coe, R. (1999), *Changes in examination grades over time: Is the same worth less?*, Paper presented at the British Educational Research Association annual conference, Brighton, September; Coe, R. (2006), "Are A levels and GCSEs getting easier?", Presentation at "Alternatives to A level and GCSE" Conference, Wellington College, 27 June.
32 This mark could be attained by answering correctly the first eleven questions on Paper 3 and the first nine questions on Paper 4.

The implications

Reduced capability for logical thinking and organisation: Pupils are emerging from school having had fewer requirements to think for themselves. In a modern globalised economy, people need increased personal capability to push themselves ahead.[33] The ability to improve wider personal capability will be the critical factor in the future economic position of individuals. "Soft skills" and the motivation and flexibility to succeed are growing in importance.[34]

Less developed toolkit: The ability to manipulate concepts and numbers has been reduced. Pupils are emerging with a less developed mathematical "toolkit" which makes them less successful at carrying out A-level courses and university courses.[35]

Difficulty in progressing to high level mathematics: The decline in standards has had a knock-on effect on A-level standards since most A-level candidates are now less well prepared at the start of their courses.[36] Moreover the changeover to modular A-levels has been accompanied by a further reduction in the mathematical capability of successful A and AS level candidates.[37]

Broader knowledge: While pupils have a less developed "toolkit", the broader curriculum means that current pupils have a wider set of knowledge, for example in statistics, which was almost entirely absent in O-level papers. Pupils are better schooled in presentation and statistics analysis – albeit at a relatively superficial level.

Less engagement with the subject: With much of the challenge and progressive understanding that mathematics can bring removed, pupils have less enjoyment in the subject due to the "tick-box" approach. This has led to declining numbers studying mathematics at later stages and a worsening perception of the subject.[38]

Less prepared for the workplace: School-leavers are less prepared for the workplace due to the poor standard of grade C GCSE mathematics.[39]

Growing inequality between state and private schools: Much of the essential rigour that is lost in the state school system continues to be respected in independent schools. This has contributed to the increasingly unhealthy divide between these two worlds. Independent schools are also free to prepare their students for international examinations such as IGCSE, which are not available to state schools; these exams often focus more strongly on core material, and so lay a better foundation for subsequent study. This in turn attracts more able mathematics teachers into the independent sector, which contributes further to better A-level results than one might otherwise expect – all of which exacerbates the long-term inequality between the two sectors. This has been translated into higher results in the independent sector.[40]

Evidence has shown that students from a higher socio-economic background are more likely to do mathematics A-level than students from a lower socio-economic background.[41]

33 Bosanquet, N. et al. (2008), *Shifting the unequal state: From public apathy to personal capability, Reform*.
34 The CBI has documented the high levels of dissatisfaction among employers with the key skills of school leavers. These are: business awareness 70 per cent; self-management 65 per cent; foreign language skills 61 per cent; general employability skills 52 per cent; knowledge of chosen job/career 52 per cent; positive attitude to work 47 per cent; basic literacy and use of English 45 per cent; and basic numeracy skills 44 per cent. See CBI (2007), *Shaping up for the future: The business vision for education and skills*.
35 Lawson, D. (2000), *Results from diagnostic testing in the 90s*, Mathematics Support Centre, Coventry University; The Engineering Council (1999), Measuring the mathematics problem.
36 London Mathematical Society (1995), *Tackling the mathematics problem*.
37 This can be seen in the decline in standards of students entering mathematics courses at degree level. See The Engineering Council (1999), *Measuring the mathematics problem*.
38 PREMA (2005), *Towards gender sensitive mathematics education: The case of England and Northern Ireland*. This study found that the majority of student response to mathematics was "lukewarm" in the sense that they found it neither boring nor exciting, challenging or unchallenging. Many, especially girls, failed to see how it would be useful for future careers due to the tick box approach that is used. See also Brown, M., Brown, P., Bibby, T. (2007), *"I would rather die": attitudes of 16-year-olds towards their future participation in mathematics*. The study found that perceived difficulty and lack of confidence are the major reasons why students don't enjoy mathematics.
39 See CBI (2006), *Working on the three Rs: Employers' priorities for functional skills in mathematics and English*.
40 In 2004, 22 per cent of GCSE maths entries in independent schools were awarded an A*, compared to 4.6 per cent nationally.
41 Vidal Rodeiro, C. L. (2007), *A-level subject choice in England: Patterns of uptake and factors affecting subject preferences*, Cambridge Assessment, p.34. Students from independent schools are almost twice as likely to do mathematics A-level as those from comprehensive schools, with 39 per cent and 22.4 per cent respectively taking the subject, according to a Cambridge Assessment survey of students. The numbers for further mathematics are even more striking, with 5.3 per cent in independent schools taking the subject compared to 1.8 per cent in comprehensive schools.

3
A lost generation of mathematicians

The changes in standards identified in the previous chapter mean that the quality of mathematics education in England has diminished in the 1970-2007 period. A further analysis of the level of actual attainment in mathematics means that it is possible to speak of a lost generation of mathematicians, and to estimate the losses both to individuals and to the economy.[42]

This is not to say that good mathematics attainment does not exist in the UK. Strong schools are sending hundreds of excellent candidates to the UK research universities. These candidates then become highly prized commodities for firms in the City of London and overseas. But the number of strong schools is limited and is disproportionately made up of independent schools.

A historic failure of attainment

The Department for Education and Skills has published data on the percentage of students entered for O-levels and GCSEs since 1951 who achieved grade C or better. This percentage began to fall in the early 1970s, only to rise in the early 1980s. With the introduction of GCSEs, a much wider group of students were entered for the exam and for a time efforts were made to align the new GCSE with the old O-level, with the result that the percentage at first fell. However, since then the percentage achieving a grade C has risen inexorably.

It is possible therefore to combine the trends in mathematics standards and 16+ attainment since the 1950s. This gives four distinct periods:

> 1950s and 1960s: rising attainment, during which the rigour of mathematics O-level was maintained.

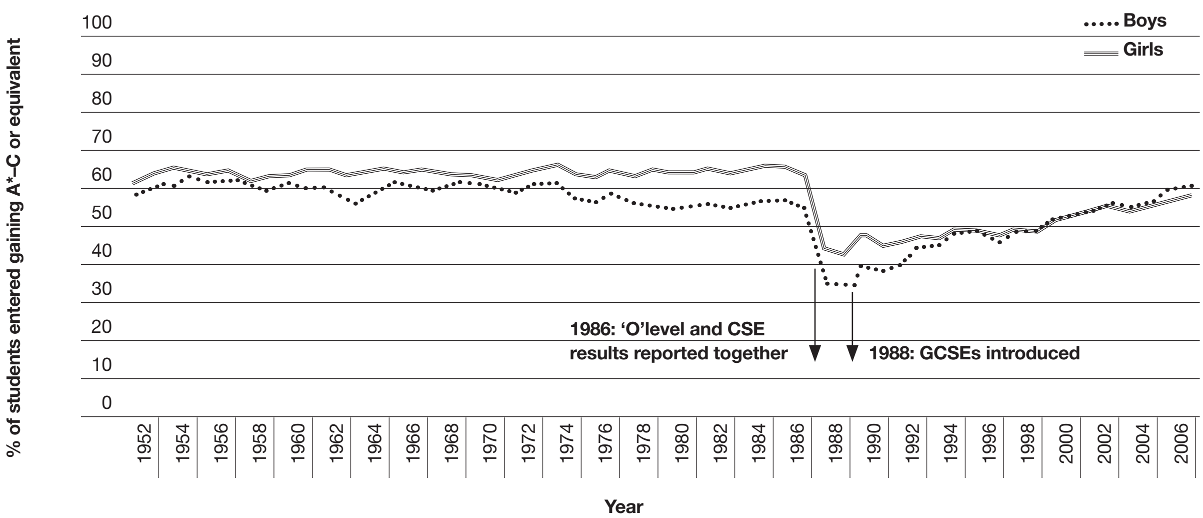
> 1970s: an end to the rise in attainment, and the beginning of the diminution of examination standards.[43]

> 1980s: a rise in attainment, accelerating sharply when the GCSE replaced the O-level.

> 1990s and 2000s: further rises in attainment coupled with further weakening of rigour.

As a result, the apparent rise in attainment over the period is highly misleading. In 2007, 55.2 per cent of pupils achieved A*-C in mathematics GCSE; but only 10 per cent or so continue to study mathematics at AS level. The historical evidence outlined above, the basic weaknesses observed among AS level students, and the experience of those teaching numerate subjects in universities all point to the fact that less than 25 per cent of 16 year olds currently master elementary mathematics to the kind of level which could be deemed acceptable.

Table 1. Per cent of students passing of those entered for O-level/GCSE (pass/entry)
Source: Department for Education and Skills (2007), *Gender and education: the evidence on pupils in England*



42 Gardiner, T. (2005), *Where will the next generation of UK mathematicians come from?*
43 Marks, J. (2002), *Standards and spending: Dispelling the spending orthodoxy*, Centre for Policy Studies, p.5.

The Table above shows the per cent of students passing of those entered for mathematics O-level/GCSE. Clearly, the results either side of 1987 are not directly comparable because the cohort increases from roughly 20 per cent of 16 year olds taking O-level to all 16 year olds taking GCSE.

Able students neglected

Britain has traditionally nurtured a remarkable number of the world's best mathematicians and scientists – as indicated by the number of Nobel prize winners and Fields' Medallists. This is no longer the case. For the last 20 years we have no longer provided the kind of foundation for the top 20-25 per cent which is needed to generate a sufficient pool of "basically competent" students, from which the small number of exceptional talents and the much larger number of technically qualified graduates might emerge. In the large-scale 2003 TIMSS international comparison, English performance was compared with a basket of 12 countries chosen by the DfES.[44] In this comparison group, 13 per cent of pupils scored at or above the "advanced benchmark" – so it is reasonable to suggest that around 20 per cent of pupils in these countries were happy working at or around this level. But just 5 per cent of English Year 9 pupils managed to score at this level. (Despite the name, these problems are in no way "advanced", in that 44 per cent of pupils in Singapore managed to perform at this level). The English performances at this level in the corresponding studies in 1995 and 1999 were almost identically poor (namely 6 per cent).

One author has explained that for traditional academic subjects, good GCSE performance is an essential step towards later study.[45] It is therefore striking that in 2007, 60,093 UK students took mathematics A-level compared to 84,744 in 1989.[46] The number of A-level mathematics entries as a proportion of total A-level entries fell by two-thirds.[47]

This is strong evidence that the changes to GCSEs have undermined later attainment. Exams have changed from being a staging-post to further study to being a series of "tick-boxes". The reduction in interest has reduced the attractiveness of the subject as well as students' ability to cope with a higher level of difficulty. Academics have also identified a decline in the standards of A-level mathematics examinations in the 1990s.[48]

More positively, the number of pupils taking further mathematics A-level is slowly increasing. This year, 7,872 students sat the subject, compared to 5,315 in 2003. This is a reminder that some effective mathematics education continues.

A divided university sector

Mathematics attainment at university has also been compromised. In interviews, current UK academics have reported that the sector has become increasingly divided with only a small group of around ten institutions offering higher quality. These institutions have themselves come under pressure. The government-led expansion in higher education in the last decade has meant that they have been in effect forced to increase undergraduate numbers.[49] This has not only reduced the average quality of their intake but also reduced their teaching resources per student. It has also meant some mathematics departments at other institutions closing.

For this reason the UK's lead over the USA and Germany in terms of mathematics graduates per head of population is no cause for complacency.

44 See Ruddock, G., Sturman, L., Schagen, I., Styles, B., Gnaldi, M., Vappula, H. (2003), *Where England stands in the Trends in International Mathematics and Science Study (TIMSS) 2003: National report for England,* NFER.

45 Vidal Rodeiro, C. L. (2007), *A-level subject choice in England: Patterns of uptake and factors affecting subject preferences*, Cambridge Assessment, p.19. "Impact of ability on subject choice was also investigated in this research since prior attainment (e.g. GCSE results) is likely to constrain choice, in particular of academic subjects. It is noticeable that the uptake increases by attainment for the traditional academic subjects: Mathematics, English Literature, Biology, Chemistry, History, Geography, Physics, Modern Foreign Languages, Further Mathematics and Music. This trend is reversed for the newer/vocational subjects: Business Studies, Media Studies, Art and Design, Sociology, Psychology or Computing."

46 Department for Children, Schools and Families.

47 1989: 12.8 per cent. 2007: 7.4 per cent.

48 However, this depends who you count. The average number of students graduating each year from serious mathematics first degree courses remained almost constant at around 4000 for decades: 4033 in 1994/5 and 4075 in 2001/2, with a degree of variation in between. In 2002/3, the definitions were changed to include many courses with limited mathematical content under "Mathematics", so that the 2002./3 figure leapt by 25 per cent to 5100.

49 Higher Education Statistics Agency. 1997: Number of students taking mathematical sciences: 19,908. Percentage of all students: 1.1. 2007: Number of students taking mathematical sciences: 33,790. Percentage of all students: 1.4.

Table 2: Number of mathematics graduates/million population[50]
Source: OECD StatsExtract, *Dataset: Graduates by field of education*

	Million population	Number of maths graduates (2005)	Number of maths graduates/ million population (2005)
USA	304	20,004	66
Germany	82	4,524	55
France	64	10,208	160
UK	60	7,143	119

Another widely-held concern is that the UK has the shortest mathematics degrees in Europe. While some courses have been extended from three to four years, this has been in order to compensate for a decline in standards in A-levels rather than an increase in the difficulty of courses.[51]

This failure lies in the education system's inability to nurture young students. As a result educational authorities find it almost impossible to recruit mathematics graduates with good degrees. One academic expert has said that the mathematics community now falls short of "reproducing itself"[52] resulting in a growing danger of the UK "becoming totally dependent on imported intellect."[53]

Attainment in the workplace

There is also strong evidence that low mathematical attainment is undermining the effectiveness of firms.

Basic skills

The Government's estimate is that 15 million adults struggle with basic mathematics.[54] The 2007 CBI/ Pertemps Employment Trends Survey showed that 50 per cent of employers were dissatisfied with the basic numeracy of school leavers.[55]

Professional level

Science and engineering businesses are already facing a recruitment crisis with industrial, energy and utility companies expecting a shortfall of up to 80 per cent this year. The Smith Report presented a number of difficulties faced by employers in recruiting appropriately qualified graduates. It concludes that the current school curriculum and qualifications framework "fails to meet the mathematical requirements of learners, fails to meet the needs and expectations of higher education and employers and fails to motivate and encourage sufficient numbers of young people to continue with the study of mathematics post-16".[56]

"Power maths"

As noted in the first chapter, the very high-value-added activities of firms in the City of London are heavily mathematics-based. In interviews for this report, one investment banker reported that of the recent recruitment of high level mathematicians, only 1 out of 8 was a UK graduate. Another has remarked that no recent mathematicians had been recruited from the UK. Several reported that they only recruited from Oxbridge in the UK, whereas broader recruitment from France, Germany and Singapore had a higher overall standard. It was widely felt that French mathematicians were particularly attractive.

50 There are no agreed international definitions of what constitutes a "mathematics graduate" so these figures can only be an estimate.
51 *Times Higher Educational Supplement* (2008), "Why we can't turn our backs on the league-table generation", 10 January. Imperial College London, for example, has turned its three-year BEng course into a four-year MEng because "half the first year is taken up with remedial teaching", according to the university's admissions chief David Robb.
52 Gardiner, T. (2005), *Where will the next generation of UK mathematicians come from?*, p. 1.
53 BBC News (2005), "Mathematics Stuck in a Downwards Spiral".
54 Department for Education and Skills (2007), *The Skills for Life survey – A national needs and impact survey of literacy, numeracy and ICT skills*. DfES Research Brief RB490.
55 CBI (2007), "School leavers dazzle with IT skills but fall short at maths and English", news release, 20 August.
56 Smith, A. (2004), *Making mathematics count*.

A lost generation

The evidence presented above shows that a decline in rigour and interest in GCSE has undermined mathematics attainment in later study and in the workplace.

Ironically, the shift to a more "relevant" mathematics curriculum was partly aimed to help the economy. Relevance has replaced rigour in an attempt to make mathematics accessible, creating "pick n' mix" mathematics where pupils dip into various subject areas able only to answer very specific questions. This has both failed to give students the tools they need to progress and turned them off the subject.

The most telling fact is the simultaneous decline in the demand of A-level mathematics and the decline in numbers. If the number of A-level mathematicians had remained constant (as a proportion of all students), there would have been an additional 430,700 over the period.

Each of these students would have earned an additional £3,080 per year due to the market premium on A-level mathematics, equating to £136,000 over their lifetime. The total gain to the economy over the period would have been over £9 billion.

Table 3: "Lost" mathematicians since 1989
Source: Department for Education and Skills; *Reform* calculations

	Number of A-level mathematicians	Ratio of mathematicians to all A-level entries	"Lost" mathematicians (cumulative)	Total lost earnings (cumulative)
1989	84,744	12.8		
2007	60,093	7.5	430,031	£9.03 billion

4

The tightening Gordian knot of political control

The political response to falling standards and attainment has been to increase political control. As a tough rigorous discipline that forms the basis of much other understanding and virtually all skilled work, mathematics has been a central focus.

Rather than using a developing global economy and a wider demand for skills to open education and liberalise its provision, education policy has been characterised by increasing central control, top down planning and producer capture. In the rapidly changing UK economy that has shifted to primarily service-based, politicians and educational authorities have found themselves persistently behind the curve.

In an attempt to address this, regulation and direction have increased forming an ever-tightening Gordian knot which, like the fable, has proved impossible to untie.[57]

The impact of this centralisation has been that those with the keenest interest in education – employers, universities, teachers and students – have found their voices unheard and their interests marginalised.

The social divide and the skills gap

Education practice in the UK needed to change to reflect the new demand for skills in the post-war economy. It was no longer acceptable for a small percentage of pupils to be educated to a high standard; in the future these skills would be required widely across the population.

Changing skills requirements – making mathematics more relevant

Politicians were keen to direct the debate about the future of skills, from Harold Wilson's 1963 "White Heat of Technology" speech to James Callaghan's "Great debate" speech at Ruskin College in 1976 where he called for "skills that could be applied in the workplace".[58] Mathematics went from a more "conceptual" subject to one that was "relevant" that could be applied to everyday situations. However the skills that politicians thought were relevant for beginners, and the consequent imposed shift, implied a reduced emphasis on core skills which are more important at this level and for subsequent work in mathematics.[59]

The battle for equality

The desire to deliver education universally meant the need to educate the weakest in the basics whilst challenging the high performers and enabling them to go on to further study. As a core subject and one that was perceived to be difficult, mathematics found itself most torn by these two requirements. Too often, instead of taking time to achieve understanding, expectations for the weakest 25 per cent were set too high in front-led lessons driven by "pitch and pace"; and the desire to design a system of the "middle 50 per cent" meant that expectations for the top 25 per cent were set too low, leaving them with inadequate foundations for subsequent mathematics.

57 The Gordian knot was alleged to be impossible to untie. The fable tells that Alexander the Great used his sword to divide it. It is frequently used as a metaphor for an intractable problem solved by a bold stroke.

58 Wilson, H. (1963), speech to the Labour Party Conference; Timmins, N. (2001), *The five giants: A biography of the welfare state*, revised edition, p. 323.

59 In recommendation 4.4 of his report [Smith, A. (2004), *Making mathematics count*], Professor Adrian Smith, a past President of the Royal Statistical Society wrote: "The inquiry recommends that there should be an immediate review... of the role and positioning of Statistics and Data Handling within the overall 14-19 curriculum. This should be informed by a recognition of the need to restore more time to the mathematics curriculum for the reinforcement of core skills, such as fluency in algebra and reasoning about geometrical properties". No such review has been undertaken.

At the same time, a more liberal approach to education confused sensitivity to the individual child with the idea that children should explore and discover mathematics for themselves as recommended in the 1967 Plowden Report.[60] At the schools where it took place, "child centred learning" meant that pupils were not schooled in the basic tools required in mathematics.[61] While many schools continued with more traditional teaching methods, there is clear evidence that it was in inner cities that this "vacuous brand of 'progressive teaching' was leaving parents unhappy with their children's schools".[62]

Changes were taking place in the structure of schools and the teaching within them; however, there was little information about the performance of each school. Concerns began to be voiced in the 1960s.[63] In 1974, the Department of Education set up an assessment of performance unit to attempt to provide evidence on standards, but it did little to help.[64]

The quest for accessibility

Pressure for universalisation continued into the 1980s and 1990s, with the GCSE, coursework and modularisation introduced to make mathematics more accessible for all pupils. Sadly the combination proved to have the opposite effect, damaging the challenge that mathematics poses, introducing the potential for abuse of the system and destroying the integrated nature of mathematics.[65]

The GCSE was introduced in 1987 and deemed suitable for all pupils. Although the Government stated that standards would not fall, in practice it proved hard to combine the competence approach of the CSE and the academic focus of the O-level.[66] The impact was a qualification that neither delivered effective basics nor a proper platform for further study.

Coursework was introduced with the GCSE (amounting to 20 per cent of the total qualification in mathematics) in order to give a fair chance to pupils who achieved less well in exams.[67] Coursework is particularly unsuitable in mathematics as it requires practice and technique rather than the acquisition of knowledge, making it open to abuse. The Qualifications and Curriculum Authority acknowledged in 2006 that coursework "does not fulfil its stated purpose", and in 2007 mathematics coursework was abolished.[68]

Modularisation at 16-19 was made universal in 2000 by dividing up A-levels into AS levels and A2 levels in order that pupils could "get part of the subject under their belts and have less pressure at the final examinations". However, it has been a destructive influence as mathematics is an integrated subject that needs to be taught sequentially. Students cannot forget one module to move on to the next, they need to understand the subject as a whole. Critics have asserted that it has made it "impossible to teach and to assess mathematics in an integrated way", making the subject "less appetising".[69] An American study by William Schmidt has shown that the top achieving countries have a progressive, "coherent" curriculum of increasing complexity as the pupils move up the school.[70] The same mistake is about to be repeated for GCSE which goes modular in 2009.

There has also been a drive for "parity of esteem" between subjects. Historically mathematics would have been considered a difficult subject and further mathematics more difficult still, a fact taken into account by employers and universities.[71] Further mathematics is now obliged to be of comparable difficulty to ordinary mathematics A-level: the QCA bureaucracy appears incapable of understanding the need for more stretching qualifications, or that they must receive correspondingly enhanced recognition (for example, enhanced UCAS points).

60 Timmins, N. (2001), *The five giants: A biography of the welfare state*, revised edition, pp. 237-8; Douglas, R., Richardson, R., Robson, S. (2003), *A better way, Reform*, p.81. "Serried rows of desks gave way to grouped tables, and the old didactic style of teaching using just 'chalk 'n talk' disappeared. Educational standards had never really appeared in the education debate hitherto. The very acquiring of an education was seen as an achievement in itself. It was only once universal education for all was achieved that the outcomes of that education came to be of interest. It marked the beginnings of a massive cultural change."
61 John Dewey was the protagonist of progressive teaching methods. See Dewey, J. (1902), *The child and the curriculum*; Dewey, J. (1899), *The school and society*.
62 Jenkins, S. (2006), *Thatcher and Sons: A revolution in three acts*.
63 A report for the Institute of Economic Affairs in 1964 stated: "It is now almost universally accepted that British education is facing a crisis." Peacock, A. T., Wiseman, J. (1964), *Education for democrats*, IEA. Perhaps the strongest impact came in 1967 in the *Black Papers*, which called for demanding tests at seven, eleven and fourteen.
64 Timmins, N. (2001), *The five giants: A biography of the welfare state*, revised edition, p. 319.
65 A recent report by the Qualifications and Curriculum Authority found that the majority of teachers believe that examinations designed to make maths more "accessible" fail to stretch the brightest students. Timmins, N. (2001), *The five giants: A biography of the welfare state*, revised edition, p. 319.
66 Timmins, N. (2001), *The five giants: A biography of the welfare state*, revised edition, p. 420.
67 The 1990, 2000 and 2006 papers examined all had the option of a coursework module worth about 20 per cent of the qualification.
68 BBC News (2006), "Move to end more GCSE coursework", 6 October.
69 BBC News (2005), "Mathematics stuck in a downwards spiral", 28 June.
70 Schmidt, W., Houang, R., Cogan, L. (2002), *A coherent curriculum: The case for mathematics*.
71 Robert Coe at the CEM Centre at the University of Durham has shown how maths GCSE is "harder" than other GCSE subjects. See Coe, R. (2006), "Are A-levels and GCSEs getting easier?", Presentation at "Alternatives to A level and GCSE" Conference, Wellington College, 27 June.

The Gordian knot of government control

Up to the 1970s the curriculum had largely been left to teachers – guided by textbooks and by the requirements of any public exams to be taken at age 16 or 18. However, as results stagnated, the Government became concerned that it was too far removed from what was going on in the classroom, and began to take steps to increase its influence.[72] Although the Conservatives flirted with market led methods such as vouchers in the early 1980s, the overall direction was towards more centralisation and government intervention.[73] The more government took control the more the parts of the mathematics system that could deliver were neutered. The unintended consequence of these government interventions is that pupil performance became inextricably linked to the Government in the minds of the electorate.

The apparatus of control

The biggest centralisation of teaching occurred with the Education Reform Act of 1988 which introduced the National Curriculum and national testing at seven, 11 and 14. In 1992, central authority was further strengthened via the establishment of the Office for Standards in Education and the introduction of league tables.

Centralisation continued under the Labour Government which came to power in 1997. The 1998 School Standards and Framework Act saw the powers of LEAs and the Department for Education and Employment extended and by 2001 each English school received 3,840 pages of instructions.[74] The Education Act of 2002 gave the Secretary of State power to issue guidance on pay and condition matters.

Reforming, decentralising policies have been introduced (such as Grant Maintained Schools and academies) but they have only applied to a small minority of schools. The current target, for example, will see academies forming only 13 per cent of all secondary schools (and around one per cent of all schools), and they have recently lost some autonomy after being bound to follow the national curriculum.[75]

The knot tightens

Now that Ministers are held more accountable for educational performance, governments have focused on examination results rather than educational outcomes. This centralisation of the education system has also reduced the contribution of mathematics professionals.[76] Those whom developments affect most, teachers and pupils, have become less able to influence developments, and shape the curriculum.

Gaming of results

Having taken much greater responsibility for examination results, politicians have an incentive to improve results, rather than to address the underlying educational performance. The incentive structure which is imposed to "deliver" what the politicians think they want then transfers this preoccupation with "results" to all involved – from bureaucrats and exam boards to teachers and parents. Grade inflation has seen results improve from year to year, while the standards achieved have often stagnated or regressed.[77] An initial focus on pupils achieving five GCSEs at grades A*-C led to a distraction from core subjects – such as maths and English. The need for schools to get students passing five GCSEs led to "gaming", where pupils were entered for easier GCSEs or qualifications which were the equivalent to several GCSEs. When English and mathematics were included in the league tables, it became clear that these "core" subjects had not seen such an increase. Much of the regulatory control is exerted by bodies connected to government. The QCA, despite recent reforms, lacks independence and tailors the demands of the qualification system to reflect government demands, rather than pupil or teacher needs.

72 Timmins, N. (2001), *The five giants: A biography of the welfare state*, revised edition, p. 322. "The post-war assumption that a steady expansion of education would produce an increasing satisfaction with education had fallen apart".
73 Timmins, N. (2001), *The five giants: A biography of the welfare state*, revised edition, pp. 417-419.
74 Hansard (2003), Col. 563W, 17 March.
75 Tice, R. (2008), *Academies: A model education? Reform*.
76 In the 1980s, the exam boards were still controlled by universities, with academics being actively involved in setting, checking and examining. The Awarding Body procedures and committees now consist almost exclusively of school teachers. One consequence is that the assessment system no longer tries to prepare school students for what is needed at university;
77 On grade inflation, see Tymms, P., Coe, R., Merrell, C. (2005), *Standards in English schools: changes since 1997 and the impact of government policies and initiatives*, CEM Centre, University of Durham.

Lack of autonomy for teachers

Government control has resulted in an over-emphasis on testing, in a bid to show that attainment is improving. Pupils in England are some of the most tested in the world, and a culture of "teaching to the test" has developed.[78]

As a result teachers have become slavish "deliverers" of the curriculum, and have increasingly lost professional involvement in the subtle, and long-term judgements required to teach mathematics well. The current generation of teachers focus on reaching quotas rather than giving students a real understanding of the subject. Professional responsibility is removed from teachers as they are restrained by targets, with a fear that failure to achieve these could put their jobs at risk.

The curriculum has also become too broad, being neither "adequately taught nor effectively assessed".[79] It is overcrowded and teachers are not given enough time to "teach", and can feel obliged to ignore important teacher-pupil relationships and pupil understanding in order to achieve national benchmarks.[80] Dr Tony Gardiner argues that this demonstrates a move from educating to "jumping through hoops".[81]

Lack of quality teachers

There is considerable evidence to suggest that there is a lack of quality mathematics teachers in the profession.[82] Professor Adrian Smith's government-commissioned *Making Mathematics Count* identified the provision of more quality teachers as an essential pre-requisite for long term improvements to post-14 mathematics education.[83]

Measures of teaching quality are difficult to obtain, with a recent Royal Society report emphasising the lack of a consistent and reliable measure.[84] However, a recent survey of those teaching mathematics showed that over 30 per cent of those teaching mathematics do not have a post A-level qualification in the subject.[85]

A fall in teaching quality affects teachers' ability to shape and deliver the curriculum and, consequently, to produce engaged students. Dr Tony Gardiner asserts that "the new breed of teachers work harder than ever to 'deliver' the curriculum", but they often have a "weaker intuitive understanding than their predecessors of the 1970s".[86]

Reducing teachers' influence at the chalkface also reduces the incentives for highly performing mathematics graduates to join the teaching profession. The Smith report found that fewer graduates who receive an upper second or first degree in mathematics choose to train as teachers than those with lower class degrees.[87]

A recommendation made by the Smith report was to ensure the continuing professional development (CPD) of mathematics teachers. The importance of CPD in improving the quality of teachers has been highlighted in numerous reports.[88]

78 James, M., Pollard, A. (2008), *Learning and teaching in primary schools: insights from TLRP*, The Primary Review.
79 Gardiner, T. (2006), *Beyond the soup kitchen: Thoughts on revising the Mathematics "Strategies/Frameworks" for England*, p.32.
80 The Smith Report, too, has highlighted that the curriculum is "not fit for purpose" and has suggested moving "newer" topics such as data handling out of the maths syllabus to another discipline in order to concentrate on "core" mathematics. Smith, A. (2004), *Making mathematics count*.
81 Gardiner, T. (2006), *Beyond the soup kitchen: Thoughts on revising the Mathematics "Strategies/Frameworks" for England*, p.3.
82 In 2003, the then DfES asserted that for a teacher to be a specialist in a subject they must have 'an adequate qualification'. The Royal Society has found that where teachers had a very good qualification in their specialist subject their teaching was excellent, where adequate it was poor. The Royal Society (2007), *A "state of the nation" report. The UK's science and mathematics teaching workforce*.
83 Smith, A. (2004), *Making mathematics count*. A McKinsey report argues that: "The quality of an education system cannot exceed the quality of its teachers." McKinsey&Company (2007), *How the world's best-performing school systems come out on top*.
84 The Royal Society (2007), *A "state of the nation" report. The UK's science and mathematics teaching workforce*.
85 Smith, A. (2004), *Making mathematics count*, p.5. The relative lack of good maths graduates means that job opportunities for them are diverse. The TDA's Performance Profiles from 2005/2006 demonstrate that of those who decided to train as mathematics teachers, 17.8 per cent of them drop out of the course before its completion. The Royal Society (2007), *A "state of the nation" report. The UK's science and mathematics teaching workforce*, p.53.
86 Gardiner, T. (2006), *Beyond the soup kitchen: Thoughts on revising the Mathematics "Strategies/Frameworks" for England*, p.2.
87 Smith, A. (2004), *Making mathematics count*.
88 Millett, A., Brown, M., Askew, M. eds. (2003), *Primary Mathematics and the Developing Professional*; McNamara, O., Jaworski, B., Rowland, T., Hodgen, J., Prestage, S. (2002), *Developing mathematics teaching and teachers*.

Universities are excluded

Universities are increasingly excluded from contributing to the development of mathematics education in schools. In the past universities played a much greater role. The old examination boards (University of London, Oxford, Cambridge, Oxford and Cambridge), used to be closely associated with the university structure in the UK. Dr Martin Stephen, the High Master at St Paul's Boys School, commented in a previous *Reform* paper: "One is hard pressed to find a university lecturer setting or marking secondary examinations, and universities seem to have little or no say in the content and difficulty of the qualifications that decide who comes to their institution."[89]

The top universities, such as Cambridge and Oxford, now set their own mathematics entrance test. This strongly suggests that the top universities no longer trust standards of mathematics A-level as a reliable indicator of a pupil's abilities.

Possibly as a response to being shut out of this process, leading universities, principally Cambridge, have helped to develop a new qualification, the Pre-U. Many leading independent schools, including Eton, Rugby and Charterhouse, are reported to be opting for this course, as it is perceived to be more rigorous than standard A-levels.[90]

Increased divide between independent and state sector

Changes imposed on state education take time to leak into the independent sector – and are often actively resisted. So independent schools are more likely to retain rigourous teaching methods and to choose examinations from outside the Government's approved canon. Independent schools are increasingly discarding the standard mathematics GCSE in favour of the IGCSE, as the latter has longer, unstructured questions (much like the O-level papers). This has resulted in a further divergence in standards and less interaction between the two sectors.[91]

89 Stephen, M. (2007), *Standards and structures: Improving the quality of teaching in English schools, Reform*.
90 *The Daily Telegraph* (2008), "State schools to offer A-level alternative", 18 April.
91 Bosanquet, N. et al. (2008), *Shifting the unequal state: From public apathy to personal capability, Reform.*

5
Winning the battle for the maths economy

The UK is being challenged by a wide variety of competitors – from established European countries to new entrants such as China and India – to produce first class mathematicians who can drive economic advance. This is particularly critical for the UK given the importance of advanced commercial activity, IT, and financial services in the economy. The UK's wider economy will rely on a much better basic level of mathematics as technological advance further removes manual and unskilled jobs. The UK must be prepared to recognise the importance of mathematics for the future.

The starting point must be to recognise the scale of the problem that has been created. Over the past 30 years the expertise that the UK had traditionally established has been squandered, with valuable resources being lost. The pre-eminence of British academic mathematics has been thrown away: British universities are more active than ever, but very few of the appointments made in recent years have been home-grown mathematicians. Our contributions to high end mathematics in the financial sector are also under threat.

Thirty years of ideological debate and politicisation of mathematics at a school level has contributed to this decline. In an attempt to make mathematics more relevant and accessible at GCSE, the level of challenge, enjoyment of the subject and attainment has been reduced. The effect has been a proportional decline in the numbers of pupils studying mathematics at A-level and university and going on to high level mathematics roles. This shortfall has in turn created a shortage in the number of adequately qualified mathematics teachers now entering schools.

The perception that mathematics is dull, difficult and "geeky" is now a widely held view in the UK. It has become acceptable to say "I'm no good at mathematics", whereas people would be ashamed to admit that they couldn't read.

This challenge is not insurmountable. Though the UK has taken the wrong path and is failing to progress, there is much retained skill both in commercial and academic fields, shown by the strength of the City of London and by the academic performance of UK universities. Equally, there are elements of the Government's policy which could form the seeds of a new policy:

- Teach First: Teach First encourages top graduates, who would not normally enter teaching, to teach for at least two years in challenging secondary schools. The scheme began in London but has been expanded to the North West and the Midlands. Currently 120 schools employ Teach First graduates. This innovative programme plays a role in bringing quality teachers to underperforming schools but remains very small in scale.[92]
- Academies: Academies are state-funded independent schools set up in areas of economic and social deprivation. There are currently 83 academies open, with a total of 400 planned. Because some of these will be sponsored by independent schools, they may offer a means of sharing the high standard of teaching in many of those schools to state pupils.[93]
- Mathematics Master Classes: Teachers from highly performing independent schools have taught classes to state school children with high ability in mathematics from under-achieving schools in the state sector.[94]

Despite their strengths, these three examples only begin to scratch the surface. What is needed is a concerted attempt over many years to change the culture in state schools. Talk about "eliminating poverty" is futile if we do not find ways to eliminate the poverty of the mind that needlessly condemns so many children to low grade jobs, or no jobs at all.

92 teachfirst.org.uk.
93 Hansard (2007), Col. 1322W, 10 July.
94 isc.co.uk.

Effective mathematics teaching requires a school and classroom ethos where teacher and pupils can slowly come to appreciate, and to enjoy, the curious mental universe of elementary mathematics. Knowing one's tables and becoming fluent in mental arithmetic develops alongside precision in the use of language and the discipline of laying out work in standard ways. These low-level procedural habits then provide a platform for tackling more interesting problems and for beginning to appreciate the remarkable coherence and power of elementary mathematics.

But first we need to convince ourselves that such talk is not merely fanciful: so we need to plan, implement and analyse tightly focused pilot projects designed to clarify how such an approach might be made to work in state schools. If the UK is to win the battle for the maths economy, real change must happen. Clearly this means a new role for government. But more importantly it means a new view and practice of mathematics, not only in education, but also in the wider society.

A new rigour

Rigour and challenge is the basis of successful mathematics teaching. The "eureka" moment experienced on the solving of a difficult problem is what motivates and spurs pupils on. A focus on rigour and challenge should not be viewed as retrograde. In a globalised economy, where more and more basic processing jobs are being undertaken by machines or those who are cheaper to employ than in a high cost country like Britain, higher skills are needed. As *Reform* authors have argued in regard to social mobility, these skills are not about following rules or processes but rather are about initiative, motivation and flexibility.[95]

Clearly the teaching profession and mathematics academics need to take the lead. Teachers need to enthuse pupils and use their own initiative to do so. Headteachers should seek to reward teachers who want to inspire pupils and teach exciting and challenging ideas.

But pupils and parents also have a vital role: to demand rigour and to reject a lower level of mathematics teaching. Parents can already make their views felt to teachers, and their "voice" will be stronger if politicians are able to increase their ability to choose.

One consequence of a rigorous approach would be an end to modularisation. Mathematics needs to be taught as an integral subject. To master advanced concepts students need to have studied the basic concepts in depth. A new commitment to rigour would see the plans to introduce a modular GCSE in 2009 scrapped as they would lead to further confusion and diminution of standards.

The capability approach

A new, capability approach applies both in education and across life. In education, it recognises that the real determinants of performance are pupils, parents, teachers and school management. In a capability approach, pupils want to study mathematics and push themselves to master the subject, and are duly rewarded for the effort required to achieve this.

Equally adults need to be sufficiently motivated to be willing to learn about new concepts and developments when they are needed and to develop these ideas independently. More and more, people need to change capability during their working lives. Colleges and universities could do more to provide facilities and open paths for adults to acquire new mathematics skills. The huge interest in IT and technology could be harnessed.

95 Bosanquet, N. et al. (2007), *Shifting the unequal state: From public apathy to personal capability, Reform.*

From geek to chic

We need to work to change the way elementary mathematics is perceived. Mathematics is hard – but not that hard. Mathematics can also be seen as interesting and challenging and is central to the way we live our lives now. The many people who enjoy Sudoku, logic problems and computer games highlight the considerable potential interest in mathematics. Moreover, the levels of performance achieved in certain other countries (from Flemish Belgium and Finland to Russia and the Far East) show that a combination of effort and effective teaching suffice for large numbers of ordinary pupils to master and enjoy the central parts of elementary mathematics.

The role of government is to recognise the importance of the challenge and to ensure a climate in which this change can happen. But first we must acknowledge, and work to correct, the neglect which has contributed to the present situation. We must then devise reward structures which make it easy for schools to take up this challenge: in particular, we must abandon the pretence that all subjects are equally important and equally demanding.

Mathematicians have been caught in a battle with government for too long. They need to be positively promoting the discipline. Steps like the UK Maths Challenge are a good start but need to be much more widely disseminated. Mathematics must move from geek to chic; from Cinderella to Queen of the Sciences.

Cutting the Gordian knot of political control

A new Alexander is needed to cut the Gordian knot of political control. The idea that Ministers take responsibility for school performance is the polar opposite of the capability approach. Instead, school performance should be a matter for pupils, their parents and the management of the school.

The only solution to the problem of the Gordian knot is a concerted programme of re-professionalising mathematics teachers currently in post, renewal of the system of public examinations which has become a corrupt offshoot of the centralised attempt to document "rising standards", followed by radical decentralisation. There have been various attempts to introduce more "challenge" recently such as the UK Maths Challenge, introduced in 1988, to enthuse pupils about mathematics. Too often, however, these innovations have been stifled by an overcentralised culture, where teachers are punished for diverting from "teaching to test".

Decentralisation also applies to examination standards. The part-independence of the QCA is a positive step forward: the regulatory side is now independent thus removing a potential conflict of interest between maintaining standards and increasing the levels of attainment. However, the examination system should become fully independent. State schools should also be allowed the same freedom to take international examinations as the independent sector. This would allow the development of rigorous examinations whether set in this country, by universities, or overseas.[96]

The value of mathematics

Over the last thirty years the UK has lost sight of the value of mathematics. The losers have been pupils, teachers, employees, employers and education Ministers. All of these groups will benefit from the recommendations above.

The alternative is a growing divide between UK achievement, key current competitors and new entrants. Mathematics is not just valuable and beautiful for its own sake, it will also be a critical driving force in the future success of the UK economy.

96 See Stephen, M. (2007), *Standards and structures: Improving the quality of teaching in English schools*, *Reform*.

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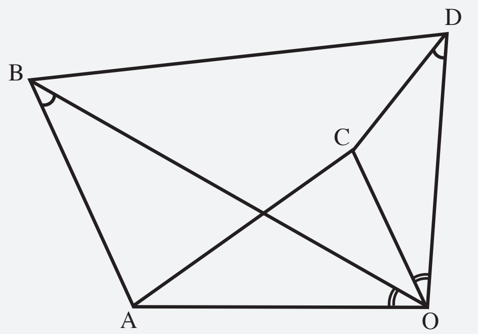
Appendix

Public Examination Questions 1951, 1960, 1970, 1980, 1990, 2000, 2006[97]

For each year, we have taken one geometry and one algebra question for comparison purposes. GCSE questions are from the higher tier, which was designed to be similar to O-level. For 1990 we also show new topics introduced.

July 1951 University of Cambridge
O-level Geometry Elementary Mathematics

12.



In the figure above, angle *ABO*=angle *CDO*,
angle *AOB*= angle *COD*

(i) Complete the statements:

$\frac{OA}{OC} = \frac{OB}{}$

$\frac{\Delta OAB}{\Delta OCD} = \frac{OA^2}{}$

(ii) Prove that the triangles *OAC*, *OBD* are similar.

July 1951 University of Cambridge
O-level Algebra Elementary Mathematics

4. Solve the equation:

$$9\left(\frac{1-x^2}{1+x^2}\right) - 7\left(\frac{2x}{1+x^2}\right) = 3.$$

July 1960
O-level Algebra Elementary Mathematics

1. (i) If $x + 2 = 0$, find the value of the expression

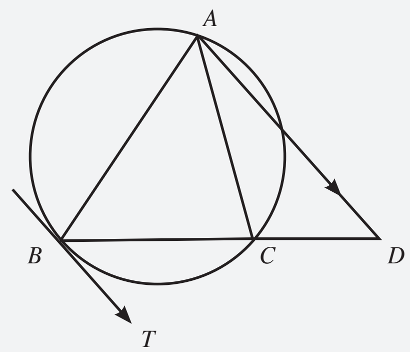
$$\frac{3(x+3)(x+1)}{x-1}$$

(ii) Divide $6a^3 - 5a^2 - 3a + 2$ by $2a - 1$

(iii) If the expression $9y^2 + kyz + 16z^2$ is a perfect square, find a value of k.

July 1960
O-level Geometry Elementary Mathematics

11.



In the figure above, *BT* is the tangent at *B* to the circle, and *AD* is parallel to *BT*. Prove that

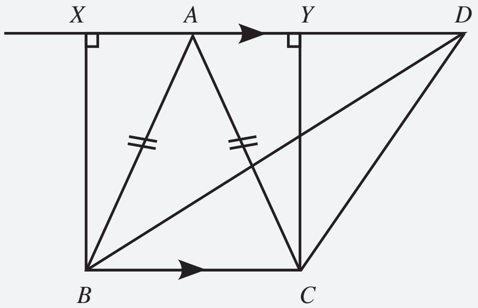
(i) the triangles *ABC*, *DBA* are similar;

(ii) $\frac{BC}{BD} = \frac{AC^2}{AD^2}$

97 With thanks to the Cambridge Assessment Archives Service

July 1970
O-level Geometry Elementary Mathematics

10.



Prove that the square on one side of an acute-angled triangle is less than the sum of the squares on the other two sides by twice the rectangle contained by one of these two sides and the projection on it of the other.

In the above figure, triangle *ABC* is isosceles with $AB = AC$. The lines *AD*, *BC* are parallel and *X*, *Y* are the feet of the perpendiculars drawn from *B* and *C* to *AD*. Complete the following statements

(i) $DC^2 = AD^2 + AC^2$ ………;
(ii) $DB^2 = AD^2 + AB^2$ ………..

Hence, or otherwise, prove that $DB^2 - DC^2 = 2AD.BC$.

July 1970
O-level Algebra Elementary Mathematics

8. (i) Solve the simultaneous equations

$$x^2 - y^2 = 28,\ x - y = 7.$$

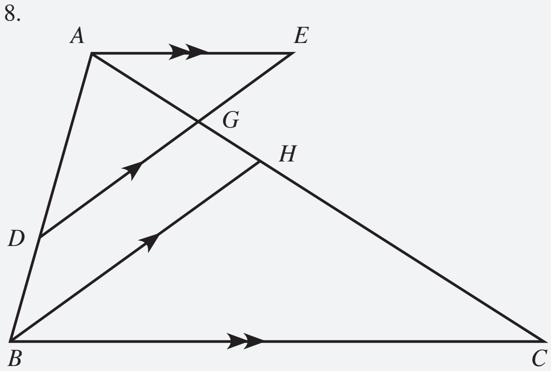
(ii) Show that $(x - 3)$ is a factor of

$$x^3 - 5x^2 - 18x + 72.$$

Without drawing a graph, find the values of x at the three points where the curve $y = x^3 - 5x^2 - 18x + 72$ meets the x-axis.

July 1980
O-level Geometry Elementary Mathematics Syllabus B

8.



In the diagram *AE* is parallel to *BC* and $AE = \frac{2}{7}BC$. The point *D* on *AB* is such that $BD = \frac{3}{7}BA$. The line *DE* meets *AC* at *G* and the line through *B* parallel to *DE* meets *AC* at *H*.

(i) Prove that $A\hat{G}E = B\hat{H}C$.
(ii) Prove that triangle *AEG* is similar to triangle *CBH*.

July 1980
O-level Algebra Elementary Mathematics Syllabus B

7. The following is an incomplete table of values for the graph of

$$y = x^2 - 3x$$

x	–2	–1	0	1	2	3	4	5
y		4		–2	–2	0	4	

(i) Calculate, and write down on your graph paper, the missing values of y.

(ii) Using a scale of 2 cm to one unit on the x-axis and 1 cm to one unit on the y-axis, draw a graph of

$$y = x^2 - 3x$$

for values of x from -2 to +5 inclusive.

(iii) Use your graph to solve the equation

$$x^2 - 3x = 5$$

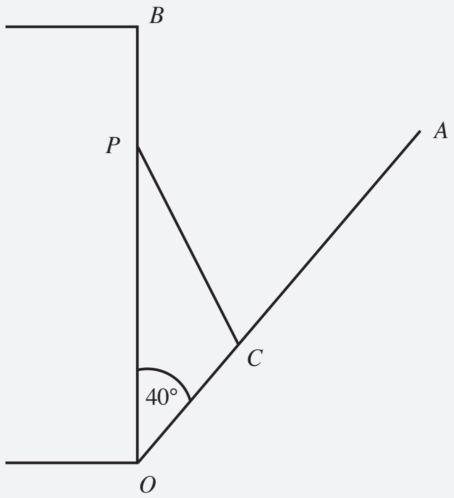
(iv) On the same diagram, draw the graph of

$$y = x + 2$$

(v) Write down and simplify the equation in x whose solutions are the values of x at the points of intersection of the two graphs.

May 1990
MEG Higher Tier Mathematics

20.



The diagram, which is drawn using a scale of 1 centimetre to represent 5 centimetres, shows the side view of a wall-cupboard with the door *OA* partly open. The door is hinged at *O* and when the cupboard is closed *A* is at *B*. The door will swing downwards until it is horizontal, when the cupboard is fully open. It is supported by a strut *PC*. *C* is fixed to the door and *P* is free to slide along *OB*.

(a) (i) Draw accurately the position of *OA* when the cupboard is fully open.

(ii) Draw accurately the locus of *A* as the door is opened.

(iii) Draw accurately the strut *PC* when the door is fully open.

(b) (i) Write down the actual length of the door, *OA*.

Answer (b) (i) ____________ cm [1]

(ii) Taking $\pi = 3.14$. Calculate the distance, correct to the nearest centimetre, moved by *A* from the position in the diagram to the fully open position.

....

....

Answer (b) (ii) ____________ cm [3]

May 1990
MEG Higher Tier Mathematics

2. (a) In a History examination, the marks range from 22 to 65. It is decided to scale the original marks using the formula

$y = 2x - 30$

where x is the original mark and y is the new mark.

(i) Calculate the lowest new mark and the highest new mark.

(ii) Express x in terms of y.

(b) In a Mathematics examination, the marks range from 24 to 84. The original marks are to be scaled so that the lowest new mark is 30 and the highest new mark is 120. The formula is to be of the type $y = ax + b$, where x is the original mark and y is the new mark. Write down two simultaneous equations for a and b. Hence find the required formula.

In 1990 new topics were introduced

May 1990
MEG Higher Tier Mathematics

21. Solve the inequality

$2x < 14 < 3x + 5.$

....

....

Answer ______________ [2]

22. The position vector of the point A is $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

that of the point B is $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ and 0 is the origin.

(a) Find $\overrightarrow{AB}$

....

....

Answer (a) ______________ [2]

(b) $\overrightarrow{BC} = 3\ \overrightarrow{OA}$

Find the coordinates of C

....

....

Answer (b) ______________ [2]

(c) The point P has coordinates (x, y). If $BP = 5$, find an equation connecting x and y.

....

....

Answer (c) ______________ [2]

May 1990
MEG Higher Tier Mathematics

6. You must use graph paper for this question.

The matrix A = $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ represents the transformation T, where

$$T: \begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

(a) Taking 2 cm to represent 1 unit on each axis and marking each axis from -3 to 3, draw and label the triangle PQR whose vertices are $P(3,0)$, $Q(1,2)$ and $R(3,-1)$.

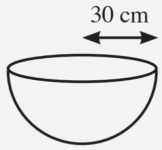
(b) The triangle PQR is mapped onto triangle $P_1Q_1R_1$ by the transformation T. Draw and label triangle $P_1Q_1R_1$ on your diagram.

(c) Describe the transformation T fully in geometrical terms.

(d) (i) Find the inverse matrix A^{-1} and the matrix A^3.
(ii) Explain the relationship between A^{-1} and A^3 in terms of transformations.

June 2000
OCR Higher Tier Mathematics

10.



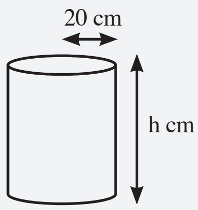
A hemispherical bowl has a radius of 30 cm.

(a) (i) Calculate the volume of the bowl. Leave your answer as a multiple of π.

....

....

Answer (a)(i) ____________ cm3 [2]



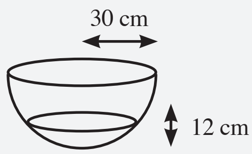
(ii) A cylinder of radius 20 cm and height h cm has the same volume as the bowl.

Calculate the value of h.

....

....

Answer (a)(ii) ____________ [2]



Water is poured into the bowl to a depth of 12cm.

(b) Calculate the radius of the surface of the water.

....

....

Answer (b) ____________ cm [3]

June 2000
OCR Higher Tier Mathematics

4. (a) Simplify $t^4 \times t^2$.

....

....

Answer (a) ____________ [1]

(b) Solve

(i) $3(x - 1) = x + 4$,

....

....

....

Answer (b)(i) $x =$ ____________ [3]

(ii) $8x + 5 > 25$.

....

....

Answer (b)(ii) ____________ [2]

(c) Factorise $4x^2 - 25$.

....

....

Answer (c) ____________ [2]

(d)(i) Factorise $x^2 + 7x + 6$.

....

....

Answer (d)(i) ____________ [2]

(ii) Hence solve the equation $x^2 + 7x + 6 = 0$.

....

....

Answer (d)(ii) $x =$ ____________ [1]

June 2006
OCR Higher Tier Mathematics

18. (a) Simplify

$$\frac{x^2 - 3x}{x^2 - 6x + 9}$$

....

....

(b) (i) Find a and b when

$x^2 + 8x + 21 = (x + a)^2 + b$

....

....

(b)(i) $a =$ ____________ $b =$ ____________

(ii) Use your answer to (b)(i) to find the minimum value of

$x^2 + 8x + 21$.

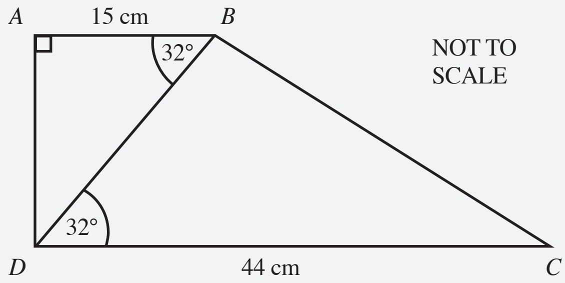
....

....

(b)(ii) ____________

June 2006
OCR Higher Tier Mathematics

12.



ABCD is a trapezium.
Angle *BAD* = 90º
Angle *BDC* = angle *ABD* = 32º
AB = 15 cm and *DC* = 44 cm.

Calculate the length of *BC*.

Give your answer to a suitable degree of accuracy.

....

....

____________ cm [6]

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